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STOCHASTIC REPAIR AND REPLACEMENT OF A STANDBY SYSTEM

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ABSTRACT

Mechanical systems deteriorate over time and do not perform according to their intended functions and eventually fail due to the failure of one or more their components or units. Failed components are either repaired or replaced depending of several factors such as cost, criticality, or reliability. Repair is perfect, minimal, or imperfect. This study assesses the performance of a standby system which upon the failure of any component is either replaced or repaired. Two models are constructed and analyzed, the first model assumes the system to be perfectly repaired after each failure, whereas in the second the failed component is either replaced or imperfectly repaired. Steady state availably is used as a performance measure.

Keywords: Availability, Imperfect Repair, Failure, Steady State

1. INTRODUCTION

Systems are maintained regularly to function properly and perform their intended design functions. Nevertheless, they malfunction after excessive usage and ultimately fail by the failure of one or more of their components. Failed components are either replaced or repaired, inexpensive and non-critical components are usually repaired while expensive and critical components are replaced. Repair could be either perfect where upon failure, the repaired component becomes as good as new; minimal repair where the component is returned to its status just before failure and imperfect repair which makes the failure of the repaired component higher prior to failure. In this article, a two component standby system is investigated.

In this system, one component is in operation and the other in cold standby. Two models of the system are presented; the first model addresses the issue of perfect repair whereas the second model examines the performance of the system where a failed component undergoes two failures before complete replacement by a new one. In the second model and upon failure of operating component, it is either replaced with probability q or is repaired with probability p. In both models, steady state availability is used as a performance indicator.

The literature is rich in research works pertinent to maintenance and repair. Barlow and Hunter (1960) used elementary renewal theory to obtain optimum policies. Nakagawa and Osaki (1975) assumed that both the working time and repair time of priority component having a general distribution while working time and repair time of the non-priority component is exponentially distributed. Some reliability indices of the system were derived using Markov renewal theory. Brown and Proschan (1983; Block *et al*., 1985; Kijima, 1989) proposed and studied many repair/replacement policies based on working age, number of repairs, repair cost and their combinations.

Zhang (2008) studied a simple repairable system with delayed repair time. He derived some important reliability indices and also obtained the optimal replacement policy N of this model. Agarwal and Mohan (2008) used the Graphical Evaluation and Review Technique (GERT) for reliability evaluation of the system to analyze an m-consecutive-k-out-of-n: F system. It is assumed that the system consists of n linearly ordered sequence of components and it fails if and only if there is at least m overlapping runs of k consecutive failed components. In this regard, the software mathematica is used for systematic computation. Haggag (2009) attempted to determine the effect of preventive maintenance on the reliability and performance of a system consisting of two dissimilar units in cold standby. The performance of the system was measured under the assumption of normal, partial failed and total failure states. The steady-state availability and cost was measured under exponential failure and repair time distributions. The results indicated that the system with preventive maintenance is better than the system without preventive maintenance.

Mujahid and Rahim (2010) examines the performance of a Preventive Maintenance Warranty (PMW) policy for repairable products with the objective of finding the optimal number of preventive maintenance actions and the length of each action and the level of maintenance needed. Additionally, failure rate and minimal repair cost, a relationship among the PM intervals is derived for a special PM case. Hanagal and Kanade (2010a) proposed replacement policy based on number of down times (or shutdown) of the repairable system. Hanagal and Kanade (2010b) also proposed optimal replacement policy based on number of down times with priority in use when the lifetime and repair time are independent. Mokaddis *et al.* (2010) analyzed the reliability of two mathematical models for an electric power system in changing outdoor weather. The first was a two-unit cold standby system and the second was two-unit warm standby system. The performance of the two systems was investigated under normal and total failure conditions. The failure times of operating/spare units and repair time of failed units were assumed to be exponentially distributed using Laplace transforms to compare the Mean Time To Failure (MTTF) of the two systems.

Hajeeh (2010) compared the performance of three configurations of two-identical component systems under imperfect repair. The steady state availability of the different configurations was derived under exponential distribution time to failure and repair times. Analysis showed that for the same components and parameters, the standby configuration performs superiorly to all the others configurations. Abdelfattah and El-Faheem (2011) attempted to improve a system by either adding hot or cold standby components, reducing the failure rate of some components, or by imperfect switches. In all the modified cases, the mean time to failure and the reliability function were derived and compared to the original system (Michlin *et al*., 2011) compared the reliability of two items by measuring the ratio of their times between failures under the assumption of exponential distribution. A methodology was presented to test the choice and dependences for determining the acceptance/rejection boundaries of such a test with pre-specified characteristics.

Oke *et al*. (2013) studied the effectiveness and cost of scheduling preventive maintenance actions for ships. In this analysis, the direct and indirect costs were included in the analysis. The main costs used are the total maintenance cost, cost of idleness, total ship idle period and total ship operation period. These costs were computed under inflation, opportunity and combined opportunity and inflation and compared with the values corresponding to maintenance cost parameter using t-test. Monte Carlo simulation is utilized to generate additional test problems. Jain (2013) examined the performance of multi-component repairable system. The steady state availability of different configurations of system is derived using supplementary variable method a recursive approach for exponential, gamma and uniform distributions of the repair time. Moreover, sensitivity analysis was conducted to evaluate the effect of system parameters on the reliability indices in addition to graphical presentation of the neuro-fuzzy results to explore the possibility of soft computing.

The maintenance problem for a simple repairable system is an important topic. In analyzing such system, many researchers and authors usually assume that the system after repair is “as good as new”. However, in real-life situation, many repairable systems deteriorate due to aging effect and the accumulative wear, tear and damages. Therefore, assuming imperfect repair where the successive operating times of the system will decrease while the consecutive repair times of the system will increase is more realistic. In this research work, the performance of a standby system subjected to imperfect repair is analyzed. It is assumed that two options are available upon failure of any component, either to replace or repair the failed component. However, the component undergoes only replacement after a pre-specified number of imperfect repairs.

2. MATERIALS AND METHODS

The current paper examines the behavior of a cold standby system subjected to imperfect repair. Standby redundancy is used in order to enhance the systems’ performance and reduce its downtime. A Standby is hot, warm and cold. In hot standby, the standby component has the same failure rate as that of the operating component, in warm standby, the failure rate of the standby component is less than the component in operation while, in cold standby system, the failure rate of the standby component is zero (i.e., the component does not fail when in standby.

The different models used in this research work are based on the following assumptions: (i) Time between failures and repair rates are exponentially distributed; (ii) all failures are statistically independent; (iii) the travel times to and from the repair facility are negligible; (iv) the system becomes as good as new after each replacement.

Several terminologies are used throughout the article, they are defined as follows:

λ_i = The i[th] failure rate of the component, i = 1,2,…, n
μ_i = The i[th] repair rate of the component, i = 1,2,…, n
π_j = The steady state probability of being at state j, j =1,2,….
a_i = Component 1 after the i[th] failure, i = 0,1,…,n; where 0 means the component is new
b_i = Component 2 after the i[th] failure, i = 0,1,…,n; where 0 means the component is new
A = Steady state availability of the system

2.1. Perfect Repair

Perfect repair brings the component to the status of as good as new; it is as replacing the failed component by a new one. A pictorial presentation of a two-component cold standby with perfect repair is given in **Fig. 1**. In this figure, the rectangular shapes represent operation states while the oval shapes represent the failed states. Hence, the states 1, 2, 3 and 4 are the operational states and the states 5 and 6 represent the states where the system has failed (down). In state 1 both components are new, A is in operation and B is in standby (bald and underlined) represent the standby component. Upon failure of A with a failure rate λ, the system moves to state 2, where component A is in repair and component B is in operation. From state 2, two transitions are likely, either to the operational state 3 by repairing component A with repair rate μ, or to the failed state 5 the failure of state B with a failure rate λ and so on.

The Chapman-Kolomogorov steady state transitional probability relationships for the Markov model for this system are as follows Equation 1:

$$\begin{aligned}
&-\lambda\pi_1 + \mu\pi_4 = 0 \\
&\lambda\pi_1 - (\lambda+\mu)\pi_2 + \mu\pi_6 = 0 \\
&\mu\pi_2 - \lambda\pi_3 = 0 \\
&\lambda\pi_3 - (\lambda+\mu)\pi_4 + \mu\pi_5 = 0 \\
&\lambda\pi_2 - \mu\pi_5 = 0 \\
&\lambda\pi_4 - \mu\pi_6 = 0
\end{aligned} \tag{1}$$

Using the relationships in (1) along with the following relationship:

$$\sum_{i=1}^{6}\pi_i = 1$$

The steady state probability of being in state 1 is derived. It has the following expression Equation 2:

$$\pi_1 = \frac{\mu^2}{2\left[\lambda^2 + \lambda\mu + \mu^2\right]} \tag{2}$$

In order to derive the expression for the system's availability, the state transitional probabilities in terms of π_1 are derived and summed. The expression after simplification and dividing the numerator and the denominator by $\lambda\mu$ (λ+μ) is as follows Equation 3:

$$A = \frac{\frac{1}{\lambda}}{\left[\frac{1}{\lambda} + \frac{\lambda}{\mu(\lambda+\mu)}\right]} \tag{3}$$

2.2. Imperfect Repair

Imperfect repair is widely used in many systems, especially the ones with expensive and no-critical components. In imperfect repair, components are repaired several times before complete replacements. In this repair type, the time between failures decreases (λ_i> = $\lambda 0_{i+1}$, I = 1,...,n) after each failure while the repair time increases (μ_{i+1} <= μ_i, i = 1,...,n). In this research work, failed components are replaced after undergoing two repairs; analytical analysis of more than two failures is too tedious. The system is presented pictorially in **Fig. 2** where the rectangular shapes are the non-failure states, whereas the oval shapes represents the failed states; the standby component is bald and underlined. In this system, the process starts form state 1, where both components are new, A is in operational state and B is standby. From state 1, the process transitions to state 2 upon the failure of component A with failure rate λ_1 and component B is operational status and component A is in repair. From state 2, the process is either moves to state 3 by the repair of components A (A_0 becomes A_1) with repair rate $p\mu_1$, to state 4 with the replacement rate $q\mu_2$, or to the failed state 17 by the failure of component B with failure rate λ_1 and so on. After the second failure of any component, it is replaced by a new one.

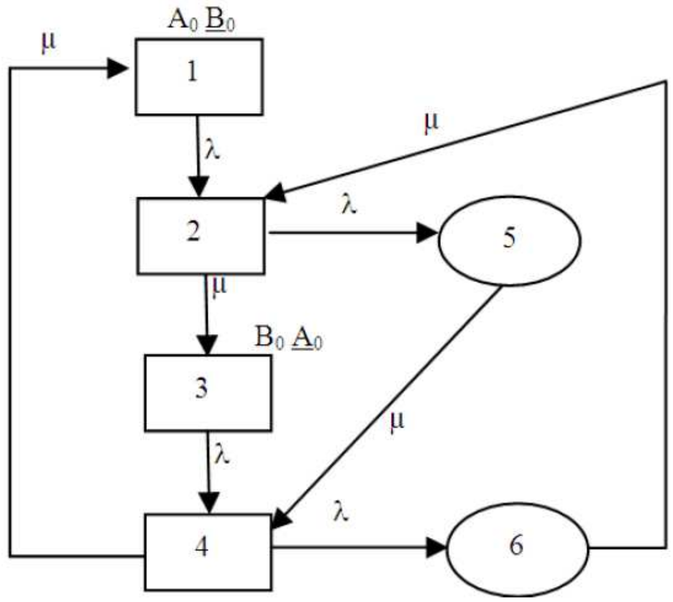

Fig. 1. A cold standby system with perfect repair

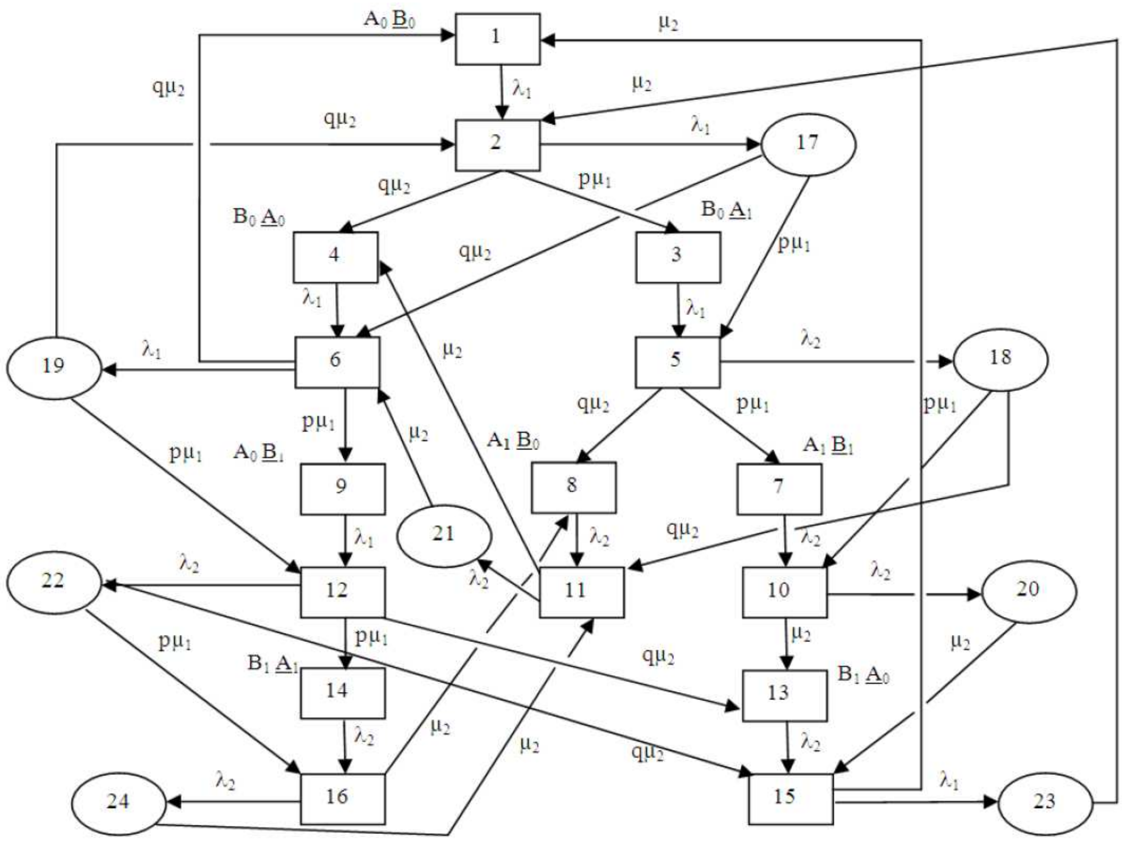

Fig. 2. Probabilistic Repair and replacement for a standby system with imperfect repair

The Chapman-Kolomogorov steady state transitional probability relationships for the Markov model for this system are as follows:

$$
\begin{aligned}
&-\lambda_1\pi_1 + q\mu_2\pi_6 + \mu_2\pi_{15} = 0\\
&\lambda_1\pi_1 - (\lambda_1 + p\mu_1 + q\mu_2)\pi_2 + q\mu_2\pi_{19} + \mu_2\pi_{23} = 0\\
&p\mu_1\pi_2 - \lambda_1\pi_3 = 0\\
&q\mu_2\pi_2 - \lambda_1\pi_4 + \mu_2\pi_{11} = 0\\
&\lambda_1\pi_3 - (\lambda_2 + p\mu_1 + q\mu_2)\pi_5 + p\mu_1\pi_{17} = 0\\
&\lambda_1\pi_4 - (\lambda_1 + p\mu_1 + q\mu_2)\pi_6 + q\mu_2\pi_{17} + \mu_2\pi_{21} = 0\\
&p\mu_1\pi_5 - \lambda_2\pi_7 = 0\\
&q\mu_2\pi_5 - \lambda_2\pi_8 + \mu_2\pi_{16} = 0\\
&p\mu_1\pi_6 - \lambda_1\pi_9 = 0\\
&\lambda_1\pi_2 - (\lambda_2 + \mu_2)\pi_{10} + p\mu_1\pi_{18} = 0\\
&\lambda_2\pi_8 - (\lambda_2 + \mu_2)\pi_{11} + q\mu_2\pi_{18} + \mu_2\pi_{24} = 0\\
&\lambda_1\pi_9 - (\lambda_2 + p\mu_1 + q\mu_2)\pi_{12} + p\mu_1\pi_{19} = 0\\
&\mu_2\pi_{10} - \lambda_2\pi_{13} + q\mu_2\pi_{12} = 0\\
&p\mu_1\pi_{12} - \lambda_2\pi_{14} = 0\\
&\lambda_2\pi_{13} - (\lambda_1 + \mu_2)\pi_{15} + \mu_2\pi_{20} + q\mu_2\pi_{22} = 0\\
&\lambda_2\pi_{14} - (\lambda_2 + \mu_2)\pi_{16} + p\mu_1\pi_{22} = 0\\
&\lambda_1\pi_2 - (p\mu_1 + q\mu_2)\pi_{17} = 0\\
&\lambda_2\pi_5 - (p\mu_1 + q\mu_2)\pi_{18} = 0\\
&\lambda_1\pi_6 - (p\mu_1 + q\mu_2)\pi_{19} = 0\\
&\lambda_2\pi_{10} - \mu_2\pi_{20} = 0\\
&\lambda_2\pi_{11} - \mu_2\pi_{21} = 0\\
&\lambda_2\pi_{12} - (p\mu_1 + q\mu_2)\pi_{22} = 0\\
&\lambda_1\pi_{15} - \mu_2\pi_{23} = 0\\
&\lambda_2\pi_{16} - \mu_2\pi_{24} = 0
\end{aligned}
\tag{4}
$$

Solving the above set of Equation in (4) and evoking the following relationship.

$\sum_{i=1}^{24}\pi_i = 1$, the probability of the system being in state 1, π_1 is obtained. It has the following structure Equation 5:

$$
\pi_1 = \frac{[q\mu_2(\lambda_1+\mu_2)(p\mu_1+q\mu_2) + \mu p\mu_1(\lambda_1+p\mu_1+q\mu_2)]}{\left\{\begin{array}{l}(\lambda_1+\mu_2)(p\mu_1+q\mu_2)(\lambda_1+p\mu_1+q\mu_2)\\ \left[1+\dfrac{p\mu_1}{(p\mu_1+q\mu_2)}\left\{1+\dfrac{\lambda_1}{\lambda_2}\left(1+\dfrac{p\mu_1}{(p\mu_1+q\mu_2)}\right)\right\}\right]\\ \dfrac{\lambda_1(\lambda_1+\mu_2)}{(\lambda_2+p\mu_1+q\mu_2)}\left[\begin{array}{l}\lambda_1(\lambda_2+p\mu_1+q\mu_2)\\ +\dfrac{\lambda_2 p\mu_1(\lambda_1+p\mu_1+q\mu_2)}{(p\mu_1+q\mu_2)}\end{array}\right]\\ +\dfrac{\lambda_1 p\mu_1(\lambda_1+\mu_2)}{\mu_2(\lambda_2+\mu_2)}\left[\lambda_1(\lambda_2+\mu_2)+\dfrac{\lambda_2 p\mu_1(\lambda_1+\mu_1)}{(p\mu_1+q\mu_2)}\right]\end{array}\right\}}
\tag{5}
$$

From the above expression, the probabilities of different states in terms of π_1 are derived. Summing, the state probabilities gives the steady availability of the system. It has the following from after manipulating and simplifying Equation 6:

$$
A = \frac{\left[\begin{array}{l}(\lambda_1+\mu_2)(\lambda_1+p\mu_1+q\mu_2)\\ \left(1+\dfrac{p\mu_1}{(p\mu_1+q\mu_2)}\right)\left(1+\dfrac{\lambda_1 p\mu_1}{\lambda_2(p\mu_1+q\mu_2)}\right)\end{array}\right]}{\left[\begin{array}{l}(\lambda_1+\mu_2)(\lambda_1+p\mu_1+q\mu_2)\\ \left(1+\dfrac{p\mu_1}{(p\mu_1+q\mu_2)}\right)\left(1+\dfrac{\lambda_1 p\mu_1}{\lambda_2(p\mu_1+q\mu_2)}\right)\\ +\dfrac{\lambda_1^2(\lambda_1+\mu_2)}{(p\mu_1+q\mu_2)}\left(1+\dfrac{p\mu_1}{\mu_2}\right)\\ +\dfrac{\lambda_1\lambda_2 p\mu_1(\lambda_1+\mu_2)}{(p\mu_1+q\mu_2)(p\mu_1+q\mu_2)}\\ \left(\dfrac{(\lambda_1+p\mu_1+q\mu_2)}{(\lambda_2+p\mu_1+q\mu_2)}+\dfrac{p\mu_1(\lambda_1+\mu_1)}{\mu_2(\lambda_2+\mu_2)}\right)\end{array}\right]}
\tag{6}
$$

3. RESULTS AND DISCUSSION

In comparing the derivation process for availability for the two models, it obvious that the process for imperfect repair is harder and lengthier than that of perfect repair case. For example, the number of states in the perfect case is six states, while in the imperfect case it is around 24 states and this number will increase as the number of imperfect repairs increases. In addition, the structure of the availability formula is more complex.

4. CONCLUSION

Analysis shows that the performance of a perfect repair is superior to that of an imperfect repair. Moreover, deriving an analytical expression for an imperfect repair system is very tedious and complex especially as the number of states increases. However, although the perfect repair option provides a higher availability, nevertheless, it is more costly because of the frequent replacement of a failed component with a new system. The imperfect repair process has several costs in addition to purchase cost such as repair cost and down time cost, but the constitute a smaller cost when compared to replacement cost.

Future work in this area should investigate the performance of a two-component system with more than two repairs in addition to systems with multiple components.

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