Original Research Paper

Direct Geometry and Cinematic to the MP-3R Systems

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Abstract: Mechatronic robotic systems are today widely used worldwide to ease human work, but especially where work is dangerous, in toxic, radioactive, chemical, explosive atmospheres, without air such as underwater or in the cosmos, or in places hard to reach the man. Robots can take the tedious repetitive work under any circumstances and they can perform a difficult operation for a long time, with no meal or rest breaks. They say the robots have stolen people's jobs. False! Robots have taken from man only those difficult jobs that baptized man and destroyed him. Robots have been created just as a prolongation of man as his real support in the faster, constantly accomplishing of difficult, repetitive, physically and mentally tired work and operations. Robots have emerged many years ago as a requirement of the automotive industry and especially that of road vehicles, especially automobiles. Meanwhile, they have diversified and branched a great deal in almost all industrial spheres. But the most important future of robots must be completely different. They have to help us conquer the cosmic space. The robot must become an extension of man in his divine mission of constructor and conqueror of the universe. Of all the types of industrial robots you use, the most common are the anthropomorphic serial robots, which is why we want to start studying and presenting the robots, with that generally common in robots, anthropomorphic serial systems. The kinematics of serial manipulators and robots will be exemplified for the 3R cinematic model to a medium difficulty system, ideal for understanding the actual phenomenon, but also for explaining the basic knowledge needed for calculating calculations and simpler or more complex systems. The fixed coordinate system was denoted by $x_0O_0y_0z_0$. The mobile systems (rigid) of the three mobile elements (1, 2, 3) have indices 1, 2 and 3. Their orientation was conveniently chosen but other orientations could be chosen. Known kinematic parameters in the direct kinematics are the absolute rotation angles of the three moving elements: φ_{10}, φ_{20}, φ_{30}, angles related to the rotation of the three actuators (electric motors) mounted in the kinematic rotation couplers. The output parameters are the three absolute coordinates x_M, y_M, z_M of point M, ie the kinematic parameters (coordinates) of the end-effector (the actuator element (the final), which can be a grasping hand, a solder tip, painted , cut, etc ...).

Keywords: Cinematic of the MP-3R Systems, Geometry, Kinematic Parameters

Introduction

Mechatronic robotic systems are today widely used worldwide to ease human work, but especially where work is dangerous, in toxic, radioactive, chemical, explosive atmospheres, without air such as underwater or in the cosmos, or in places hard to reach the man.

Robots can take the tedious repetitive work under any circumstances and they can perform a difficult operation for a long time, with no meal or rest breaks. They say the robots have stolen people's jobs. False! Robots have taken from man only those difficult jobs that baptized man and destroyed him. Robots have been created just as a prolongation of man as his real support in the faster, constantly accomplishing of difficult, repetitive, physically and mentally tired work and operations. Robots have emerged many years ago as a requirement of the automotive industry and especially that of road vehicles, especially automobiles. Meanwhile, they have diversified and branched a great deal in almost all industrial spheres. But the most important future of robots must be completely different. They have to help us conquer the cosmic space. The robot must become an extension of man in his divine mission of constructor and conqueror of the universe. Of all the types of industrial robots you use, the most common are the anthropomorphic serial robots, which is why we want to start studying and presenting the robots, with that generally common in robots, anthropomorphic serial systems (Antonescu and Petrescu, 1985; 1989; Antonescu *et al*., 1985a-b, 1986; 1987; 1988; 1994; 1997; 2000a-b, 2001; Aversa *et al*., 2017a-e, 2016a-o; Berto *et al*., 2016a-d; Cao *et al*., 2013; Dong *et al*., 2013; Comanescu, 2010; Franklin, 1930; He *et al*., 2013; Lee, 2013; Lin *et al*., 2013; Liu *et al*., 2013; Mirsayar *et al*., 2017; Padula and Perdereau, 2013; Perumaal and Jawahar, 2013; Petrescu, 2011; 2015a-b; Petrescu and Petrescu, 1995a-b; 1997a-c; 2000a-b; 2002a-b; 2003; 2005a-e; 2011; 2012a-b; 2013a-b; 2016a-c; Petrescu *et al*., 2009; 2016; 2017a-l).

Materials and Methods

The kinematics of serial manipulators and robots will be exemplified for the 3R cinematic model (Figure 1), a medium difficulty system, ideal for understanding the actual phenomenon, but also for explaining the basic knowledge needed for calculating calculations and simpler or more complex systems.

The fixed coordinate system was denoted by $x_0O_0y_0z_0$. The mobile systems (rigid) of the three mobile elements (1, 2, 3) have indices 1, 2 and 3. Their orientation was conveniently chosen but other orientations could be chosen. Known kinematic parameters in the direct kinematics are the absolute rotation angles of the three moving elements: φ_{10}, φ_{20}, φ_{30}, angles related to the rotation of the three actuators (electric motors) mounted in the kinematic rotation couplers. The output parameters are the three absolute coordinates x_M, y_M, z_M of point M, i.e., the kinematic parameters (coordinates) of the end-effector (the actuator element (the final), which can be a grasping hand, a solder tip, painted , cut, etc ...).

To begin with, write the vector matrix (A_{01}) to change the coordinates of the origin of the coordinate system by translating from O_0 to O_1, the axes remain parallel to themselves at all times (1):

$$A_{01} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} \quad (1)$$

Next, write the T_{01} rotation matrix of the system $x_1O_1y_1z_1$ to the system $x_0O_0y_0z_0$, (this is a square matrix 3×3), (2):

$$T_{01}\begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} \cos\phi_{10} & -\sin\phi_{10} & 0 \\ \sin\phi_{10} & \cos\phi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

On the first column (belonging to the coordinates of O_1x_1) the coordinates of the O_1x_1 unit vector the axes of the old system $x_0O_0y_0z_0$ are passed; it is practically the projections of the O_1x_1 unit vector on the axes of the old $x_0O_0y_0z_0$ coordinate system translated into O_1 (but not rotated, thus only the actual rotation without translation) occurs (3):

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \quad (3)$$

On the second column of the T_{01} matrix, the coordinates of the O_1y_1 axis unit vector the axes of the old $x_0O_0y_0z_0$ system are translated into O_1 without rotation (basically the coordinates of this unit vector the old translational but non-rotating reference axes), (4):

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix} \quad (4)$$

On the third column of the T_{01} matrix, the coordinates of the O_1z_1 axis unit vector the axes of the old system $x_0O_0y_0z_0$ are translated into O_1 without rotation (basically the coordinates of this unit vector the old referenced but non-routed reference axes), (5):

$$\begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \end{bmatrix} \quad (5)$$

In the chosen case, the unit vector of O_1x_1 (the unit vector always has module (1) has the following coordinates compared to the old axle system $x_0O_0y_0z_0$ translated into O_1 without rotation (6):

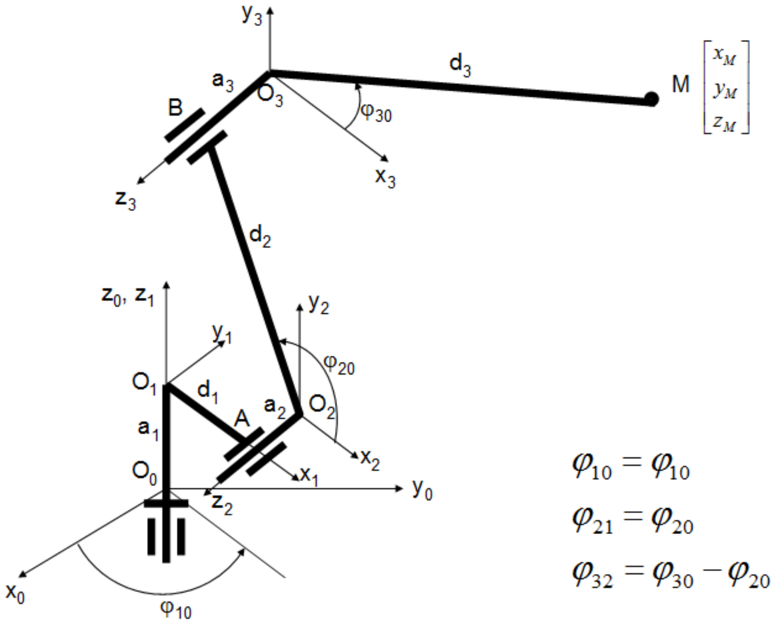

Fig. 1: The geometry and cinematic of a MP-3R

$$\begin{bmatrix} \alpha_x = 1\cdot\cos\phi_{10} = \cos\phi_{10} \\ \alpha_y = 1\cdot\sin\phi_{10} = \sin\phi_{10} \\ \alpha_z = 1\cdot\cos 90^0 = 1\cdot 0 = 0 \end{bmatrix} \quad (6)$$

O_1y_1's unit vector has the following coordinates in relation to the old $x_0O_0y_0z_0$ axis system (7):

$$\begin{bmatrix} \beta_x = -1\cdot\sin\phi_{10} = -\sin\phi_{10} \\ \beta_y = 1\cdot\cos\phi_{10} = \cos\phi_{10} \\ \beta_z = 1\cdot\cos 90^0 = 1\cdot 0 = 0 \end{bmatrix} \quad (7)$$

O_1z_1's unit vector has the following coordinates in relation to the old $x_0O_0y_0z_0$ axis system translated into O_1 without rotation (8):

$$\begin{bmatrix} \gamma_x = 1\cdot\cos 90^0 = 1\cdot 0 = 0 \\ \gamma_y = 1\cdot\cos 90^0 = 1\cdot 0 = 0 \\ \gamma_z = 1\cdot\cos 0^0 = 1\cdot 1 = 1 \end{bmatrix} \quad (8)$$

See matrix T_{01} obtained (relationship 2).

The transition from the $x_1O_1y_1z_1$ system to the $x_2O_2y_2z_2$ coordinate system is done in two distinct stages. The first is a translation of the whole system such that (the axes being parallel to themselves) central O_1 to move to O_2; then the second stage takes place in which there is a rotation of the system rotating axes and the center O remains permanently fixed. The system translation from 1 to 2 is marked by the column vector type A_{12} (9):

$$A_{12} = \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} \quad (9)$$

On the old O_1x_1 axis, O_2 was translated with d_1, O_1y_1, O_2 was translated with a_2 and O_2z_1 was not translated.

O_2x_2's unit vector, the $x_1O_1y_1z_1$ system (translated, but not rotated) coordinates (10):

$$\alpha_x = 1; \quad \alpha_y = 0; \quad \alpha_z = 0 \quad (10)$$

O_2y_2's unit vector has in relation to the $x_1O_1y_1z_1$ system translated into O_2 (not rotated) the coordinates (11):

$$\beta_x = 0; \beta_y = 0; \beta_z = 1 \quad (11)$$

Because now O_2y_2, was taken the new O_1z_1 axis:

The O_2z_2 unit vector has, in relation to the $x_1O_1y_1z_1$ system, translated into O_2 (not rotated) the coordinates (12):

$$\gamma_x = 0; \gamma_y = -1; \gamma_z = 0 \quad (12)$$

Since the O_2z_2 axis took the place of the O_1y_1 axis, it was of the opposite direction.

The square of the transfer (rotation) is written (13):

$$T_{12} = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad (13)$$

The transition from the $x_2O_2y_2z_2$ system to the $x_3O_3y_3z_3$ coordinate system is also done in two distinct stages, one translation and one rotation.

O_2 translates into O_3 (the axes are kept parallel to themselves), (14):

$$A_{23} = \begin{bmatrix} d_2 \cdot \cos\phi_{20} \\ d_2 \cdot \sin\phi_{20} \\ -a_3 \end{bmatrix} \tag{14}$$

Then O_3 stands and the axes rotate. The unit vector of O3×3 has, in relation to the axes system $x_2O_2y_2z_2$, translated in O_3 (unrotated) the coordinates α (15):

$$\alpha_x = 1; \alpha_y = 0; \alpha_z = 0 \tag{15}$$

O_3y_3's unit vector the $x_2O_2y_2z_2$ axis system is translated into O_3 (without to be rotated) coordinates β (16):

$$\beta_x = 0; \beta_y = 1; \beta_z = 0 \tag{16}$$

O_3z_3's unit vector the $x_2O_2y_2z_2$ axis system translated into O_3 (not rotated) coordinates γ (17):

$$\gamma_x = 0; \gamma_y = 0; \gamma_z = 1 \tag{17}$$

Practically, the $x_3O_3y_3z_3$ did not turn at all at all against the $x_2O_2y_2z_2$ system (from 2 to 3 only a translation took place). The matrix of rotation in this case is the unit matrix (18):

$$T_{23} \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

The vector matrix (column) that positions the M point in the $x_3O_3y_3z_3$ coordinate system is written (19):

$$X_{3M} = \begin{bmatrix} x_{3M} \\ y_{3M} \\ z_{3M} \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} \tag{19}$$

The coordinates of the point M in the system (2) $x_2O_2y_2z_2$ (i.e., towards it) are obtained by a matrix transformation of the form (20):

$$X_{2M} = A_{23} + T_{23} \cdot X_{3M} \tag{20}$$

Perform the matrix product first (21):

$$T_{23} \cdot X_{3M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} \tag{21}$$

Calculate then X_{2M} (22):

$$\begin{aligned} X_{2M} &= A_{23} + T_{23} \cdot X_{3M} \\ &= \begin{bmatrix} d_2 \cdot \cos\phi_{20} \\ d_2 \cdot \sin\phi_{20} \\ -a_3 \end{bmatrix} + \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \\ -a_3 \end{bmatrix} \end{aligned} \tag{22}$$

The coordinates of the M point in (against) the system (1) $x_1O_1y_1z_1$ are obtained as follows (23-25):

$$X_{1M} = A_{12} + T_{12} \cdot X_{2M} \tag{23}$$

$$\begin{aligned} &T_{12} \cdot X_{2M} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \\ -a_3 \end{bmatrix} \\ &= \begin{bmatrix} d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ a_3 \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \end{aligned} \tag{24}$$

$$\begin{aligned} X_{1M} &= A_{12} + T_{12} \cdot X_{2M} \\ &= \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} + \begin{bmatrix} d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ a_3 \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \\ &= \begin{bmatrix} d_1 + d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ a_2 + a_3 \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \end{aligned} \tag{25}$$

The coordinates of point M in the fixed system $x_0O_0y_0z_0$ are written (26-28):

$$X_{0M} = A_{01} + T_{01} \cdot X_{1M} \tag{26}$$

$$\begin{aligned} &T_{01} \cdot X_{1M} = \\ &\cdot \begin{bmatrix} d_3 \cdot \sin\phi_{30} & \sin\phi_{10} & 0 \\ \sin\phi_{10} & \cos\phi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_1 + d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ a_2 + a_3 \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \end{aligned} \tag{27a}$$

$$\begin{aligned} &T_{01} \cdot X_{1M} = \\ &\begin{bmatrix} (d_1 + d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30}) \\ \cdot \cos\phi_{10} - (a_2 + a_3) \cdot \sin\phi_{10} \\ (d_1 + d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30}) \\ \cdot \sin\phi_{10} + (a_2 + a_3) \cdot \cos\phi_{10} \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \end{aligned} \tag{27b}$$

$$X_{0M} = A_{01} + T_{01} \cdot X_{1M} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} + \begin{bmatrix} (d_1 + d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30}) \\ \cdot\cos\phi_{10} - (a_2 + a_3) \cdot \sin\phi_{10} \\ (d_1 + d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30}) \\ \cdot\sin\phi_{10} + (a_2 + a_3) \cdot \cos\phi_{10} \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} = \begin{bmatrix} (d_1 + d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30}) \\ \cdot\cos\phi_{10} - (a_2 + a_3) \cdot \sin\phi_{10} \\ (d_1 + d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30}) \\ \cdot\sin\phi_{10} + (a_2 + a_3) \cdot \cos\phi_{10} \\ a_1 + d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \tag{28}$$

X_{0M} takes the form of (29):

$$X_{0M} = \begin{bmatrix} x_M \\ y_M \\ z_M \end{bmatrix} = \begin{bmatrix} d_1 \cdot \cos\phi_{10} - a_2 \cdot \sin\phi_{10} + d_2 \cdot \cos\phi_{20} \cdot \cos\phi_{10} \\ -a_3 \cdot \sin\phi_{10} + d_3 \cdot \cos\phi_{30} \cdot \cos\phi_{10} \\ d_1 \cdot \sin\phi_{10} + a_2 \cdot \cos\phi_{10} + d_2 \cdot \cos\phi_{20} \cdot \sin\phi_{10} \\ +a_3 \cdot \cos\phi_{10} + d_3 \cdot \cos\phi_{30} \cdot \sin\phi_{10} \\ a_1 + d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \tag{29}$$

Results

The same calculations will be further pursued by a direct method, taking into account the matrix calculations (30a):

$$\begin{aligned} X_{0M} &= A_{01} + T_{01} \cdot X_{1M} = A_{01} + T_{01} \cdot (A_{12} + T_{12} \cdot X_{2M}) = \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot X_{2M} = \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot (A_{23} + T_{23} \cdot X_{3M}) = \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot A_{23} + T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} \end{aligned} \tag{30a}$$

The relationship is retained (30b):

$$X_{0M} = A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot A_{23} + T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} \tag{30b}$$

Perform matrix products in the expression (30') while remaining in the form of a matrix amount (31-36):

$$T_{01} \cdot A_{12} = \begin{bmatrix} \cos\phi_{10} & -\sin\phi_{10} & 0 \\ \sin\phi_{10} & \cos\phi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} = \begin{bmatrix} d_1 \cdot \cos\phi_{10} - a_2 \cdot \sin\phi_{10} \\ d_1 \cdot \sin\phi_{10} + a_2 \cdot \cos\phi_{10} \\ 0 \end{bmatrix} \tag{31}$$

$$T_{01} \cdot T_{12} = \begin{bmatrix} \cos\phi_{10} & -\sin\phi_{10} & 0 \\ \sin\phi_{10} & \cos\phi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} \\ \sin\phi_{10} & 0 & -\cos\phi_{10} \\ 0 & 1 & 0 \end{bmatrix} \tag{32}$$

$$T_{01} \cdot T_{12} \cdot A_{23} = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} \\ \sin\phi_{10} & 0 & -\cos\phi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_2 \cdot \cos\phi_{20} \\ d_2 \cdot \sin\phi_{20} \\ -a_3 \end{bmatrix} = \begin{bmatrix} d_2 \cdot \cos\phi_{10} \cdot \cos\phi_{20} - a_3 \cdot \sin\phi_{10} \\ d_2 \cdot \sin\phi_{10} \cdot \cos\phi_{20} + a_3 \cdot \cos\phi_{10} \\ d_2 \cdot \sin\phi_{20} \end{bmatrix} \tag{33}$$

$$T_{01} \cdot T_{12} \cdot T_{23} = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} \\ \sin\phi_{10} & 0 & -\cos\phi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} \\ \sin\phi_{10} & 0 & -\cos\phi_{10} \\ 0 & 1 & 0 \end{bmatrix} \tag{34}$$

$$T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} \\ \sin\phi_{10} & 0 & -\cos\phi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\phi_{10} & . & \cos\phi_{30} \\ d_3 \cdot \sin\phi_{10} & . & \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} & & \end{bmatrix} \tag{35}$$

$$X_{0M} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} + \begin{bmatrix} d_1 \cdot \cos\phi_{10} - a_2 \cdot \sin\phi_{10} \\ d_1 \cdot \sin\phi_{10} + a_2 \cdot \cos\phi_{10} \\ 0 \end{bmatrix} + \begin{bmatrix} d_2 \cdot \cos\phi_{10} \cdot \cos\phi_{20} - a_3 \cdot \sin\phi_{10} \\ d_2 \cdot \sin\phi_{10} \cdot \cos\phi_{20} + a_3 \cdot \cos\phi_{10} \\ d_2 \cdot \sin\phi_{20} \end{bmatrix} + \begin{bmatrix} d_3 \cdot \cos\phi_{10} \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{10} \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \end{bmatrix} = \begin{bmatrix} x_M \\ y_M \\ z_M \end{bmatrix} = \begin{bmatrix} d_1 \cdot \cos\phi_{10} - a_2 \cdot \sin\phi_{10} + d_2 \cdot \cos\phi_{20} \cdot \cos\phi_{10} \\ -a_3 \cdot \sin\phi_{10} + d_3 \cdot \cos\phi_{30} \cdot \cos\phi_{10} \\ d_1 \cdot \sin\phi_{10} + a_2 \cdot \cos\phi_{10} + d_2 \cdot \cos\phi_{20} \cdot \sin\phi_{10} \\ +a_3 \cdot \cos\phi_{10} + d_3 \cdot \cos\phi_{30} \cdot \sin\phi_{10} \\ a_1 + d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \tag{36}$$

Discussion

Direct kinematics obtains the Cartesian coordinates x_M, y_M, z_M of the point M (the end effector) according to the three independent angular displacements φ_{10}, φ_{20}, φ_{30}, obtained with actuators (37-38).

$$\begin{cases} x_M = f_x(\phi_{10}, \phi_{20}, \phi_{30}) \\ y_M = f_y(\phi_{10}, \phi_{20}, \phi_{30}) \\ z_M = f_z(\phi_{10}, \phi_{20}, \phi_{30}) \end{cases} \tag{37}$$

$$\begin{cases} x_M = d_1 \cdot \cos\phi_{10} - a_2 \cdot \sin\phi_{10} + d_2 \cdot \cos\phi_{20} \cdot \cos\phi_{10} \\ -a_3 \cdot \sin\phi_{10} + d_3 \cdot \cos\phi_{30} \cdot \cos\phi_{10} \\ y_M = d_1 \cdot \sin\phi_{10} + a_2 \cdot \cos\phi_{10} + d_2 \cdot \cos\phi_{20} \cdot \sin\phi_{10} \\ +a_3 \cdot \cos\phi_{10} + d_3 \cdot \cos\phi_{30} \cdot \sin\phi_{10} \\ z_M = a_1 + d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{cases} \tag{38}$$

The calculations are made with absolute angular displacements, but actuator displacements do not all coincide with the independent ones. They are thus determined (39):

$$\begin{aligned} &\phi_{10} = \phi_{10} \\ &\phi_{21} = \phi_{20} \\ &\phi_{32} = \phi_{30} - \phi_{20} \end{aligned} \tag{39}$$

The first two relative rotations of the actuators coincide with the independent rotation (used in the calculations), but the third relative rotation of the last actuator is obtained as a difference between two absolute rotations.

Speeds and accelerations are obtained by deriving relationships (38) over time.

Conclusion

The work presents an analytical method for determination of cinematic parameters in a 3R robotics module.

An exact algebraic, matrix, calculation method is presented.

Direct kinematics obtains the Cartesian coordinates x_M, y_M, z_M of the point M (the end effector) according to the three independent angular displacements φ_{10}, φ_{20}, φ_{30}, obtained with actuators (37-38).

The calculations are made with absolute angular displacements, but actuator displacements do not all coincide with the independent ones. They are thus determined (39).

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Author's Contributions

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References

Antonescu, P. and F. Petrescu, 1985. An analytical method of synthesis of cam mechanism and flat stick. Proceedings of the 4th International Symposium on Theory and Practice of Mechanisms, (TPM' 85), Bucharest.

Antonescu, P. and F. Petrescu, 1989. Contributions to kinetoplast dynamic analysis of distribution mechanisms. SYROM'89, Bucharest.

Antonescu, P., M. Oprean and F. Petrescu, 1985a. Contributions to the synthesis of oscillating cam mechanism and oscillating flat stick. Proceedings of the 4th International Symposium on Theory and Practice of Mechanisms, (TPM' 85), Bucharest.

Antonescu, P., M. Oprean and F. Petrescu, 1985b. At the projection of the oscillate cams, there are mechanisms and distribution variables. Proceedings of the 5th Conference of Engines, Automobiles, Tractors and Agricultural Machines, Engines and Automobiles, (MEA' 85), Brasov.

Antonescu, P., M. Oprean and F. Petrescu, 1986. Designing the profile of the rotating camshaft acting on the oscillating plate with misalignment. Proceedings of the 3rd National Computer-aided Design Symposium in the Field of Mechanics and Machine Organs, (FMR' 86), Brasov.

Antonescu, P., M. Oprean and F. Petrescu, 1987. Dynamic analysis of the cam distribution mechanisms. Proceedings of the 7th National Symposium on Industrial Robots and Space Mechanisms, (RSM' 87), Bucharest.

Antonescu, P., M. Oprean and F. Petrescu, 1988. Analytical synthesis of Kurz profile, rotating the flat cam. Machine Building Magazine Bucharest.

Antonescu, P., F. Petrescu and O. Antonescu, 1994. Contributions to the synthesis of the rotating cam mechanism and the tip of the balancing tip. Brasov.

Antonescu, P., F. Petrescu and D. Antonescu, 1997. Geometrical synthesis of the rotary cam and balance tappet mechanism. SYROM Bucharest, 3: 23-23.

Antonescu, P., F. Petrescu and O. Antonescu, 2000a. Contributions to the synthesis of the rotary disc-cam profile. Proceedings of the 8th International Conference on the Theory of Machines and Mechanisms, (TMM' 00), Liberec, Czech Republic, pp: 51-56.

Antonescu, P., F. Petrescu and O. Antonescu, 2000b. Synthesis of the rotary cam profile with balance follower. Proceedings of the 8th Symposium on Mechanisms and Mechanical Transmissions, (MMT' 00), Timişoara, pp: 39-44, 2000.

Antonescu, P., F. Petrescu and O. Antonescu, 2001. Contributions to the synthesis of mechanisms with rotary disc-cam. Proceedings of the 8th IFToMM International Symposium on Theory of Machines and Mechanisms, (TMM' 01), Bucharest, ROMANIA, pp: 31-36.

Aversa, R., R.V.V. Petrescu, A. Apicella and F.I.T. Petrescu, 2017a. Nano-diamond hybrid materials for structural biomedical application. Am. J. Biochem. Biotechnol.

Aversa, R., R.V. Petrescu, B. Akash, R.B. Bucinell and J.M. Corchado *et al*., 2017b. Kinematics and forces to a new model forging manipulator. Am. J. Applied Sci., 14: 60-80.

Aversa, R., R.V. Petrescu, A. Apicella, F.I.T. Petrescu and J.K. Calautit *et al*., 2017c. Something about the V engines design. Am. J. Applied Sci., 14: 34-52.

Aversa, R., D. Parcesepe, R.V.V. Petrescu, F. Berto and G. Chen *et al*., 2017d. Processability of bulk metallic glasses. Am. J. Applied Sci., 14: 294-301.

Aversa, R., R.V.V. Petrescu, B. Akash, R.B. Bucinell and J.M. Corchado *et al*., 2017e. Something about the balancing of thermal motors. Am. J. Eng. Applied Sci.

Aversa, R., F.I.T. Petrescu, R.V. Petrescu and A. Apicella, 2016a. Biomimetic FEA bone modeling for customized hybrid biological prostheses development. Am. J. Applied Sci., 13: 1060-1067. DOI: 10.3844/ajassp.2016.1060.1067

Aversa, R., D. Parcesepe, R.V. Petrescu, G. Chen and F.I.T. Petrescu *et al*., 2016b. Glassy amorphous metal injection molded induced morphological defects. Am. J. Applied Sci., 13: 1476-1482.

Aversa, R., R.V. Petrescu, F.I.T. Petrescu and A. Apicella, 2016c. Smart-factory: Optimization and process control of composite centrifuged pipes. Am. J. Applied Sci., 13: 1330-1341.

Aversa, R., F. Tamburrino, R.V. Petrescu, F.I.T. Petrescu and M. Artur *et al*., 2016d. Biomechanically inspired shape memory effect machines driven by muscle like acting NiTi alloys. Am. J. Applied Sci., 13: 1264-1271.

Aversa, R., E.M. Buzea, R.V. Petrescu, A. Apicella and M. Neacsa *et al*., 2016e. Present a mechatronic system having able to determine the concentration of carotenoids. Am. J. of Eng. and Applied Sci., 9: 1106-1111.

Aversa, R., R.V. Petrescu, R. Sorrentino, F.I.T. Petrescu and A. Apicella, 2016f Hybrid ceramo-polymeric nanocomposite for biomimetic scaffolds design and preparation. Am. J. Eng. Applied Sci., 9: 1096-1105.

Aversa, R., V. Perrotta, R.V. Petrescu, C. Misiano and F.I.T. Petrescu *et al*., 2016g. From structural colors to super-hydrophobicity and achromatic transparent protective coatings: Ion plating plasma assisted TiO_2 and SiO_2 Nano-film deposition. Am. J. Eng. Applied Sci., 9: 1037-1045.

Aversa, R., R.V. Petrescu, F.I.T. Petrescu and A. Apicella, 2016h. Biomimetic and evolutionary design driven innovation in sustainable products development. Am. J. Eng. Applied Sci., 9: 1027-1036.

Aversa, R., R.V. Petrescu, A. Apicella and F.I.T. Petrescu, 2016i. Mitochondria are naturally micro robots - a review. Am. J. Eng. Applied Sci., 9: 991-1002.

Aversa, R., R.V. Petrescu, A. Apicella and F.I.T. Petrescu, 2016j. We are addicted to vitamins C and E-A review. Am. J. Eng. Applied Sci., 9: 1003-1018.

Aversa, R., R.V. Petrescu, A. Apicella and F.I.T. Petrescu, 2016k. Physiologic human fluids and swelling behavior of hydrophilic biocompatible hybrid ceramo-polymeric materials. Am. J. Eng. Applied Sci., 9: 962-972.

Aversa, R., R.V. Petrescu, A. Apicella and F.I.T. Petrescu, 2016l. One can slow down the aging through antioxidants. Am. J. Eng. Applied Sci., 9: 1112-1126.

Aversa, R., R.V. Petrescu, A. Apicella and F.I.T. Petrescu, 2016m. About homeopathy or ≪Similia Similibus Curentur≫. Am. J. Eng. Applied Sci., 9: 1164-1172.

Aversa, R., R.V. Petrescu, A. Apicella and F.I.T. Petrescu, 2016n. The basic elements of life's. Am. J. Eng. Applied Sci., 9: 1189-1197.

Aversa, R., F.I.T. Petrescu, R.V. Petrescu and A. Apicella, 2016o. Flexible stem trabecular prostheses. Am. J. Eng. Applied Sci., 9: 1213-1221.

Berto, F., R.V.V. Petrescu and F.I.T. Petrescu, 2016a. A review of recent results on 3D effects. Am. J. Eng. Applied Sci., 9: 1247-1260.

Berto, F., R.V.V. Petrescu and F.I.T. Petrescu, 2016b. Three-dimensional in bonded joints: A short review. Am. J. Eng. Applied Sci., 9: 1261-1268.

Berto, F., A. Gagani, R.V.V. Petrescu and F.I.T. Petrescu, 2016c. Key-hole notches in isostatic graphite: A review of some recent data. Am. J. Eng. Applied Sci., 9: 1292-1300.

Berto, F., A. Gagani, R. Aversa, R.V.V. Petrescu and A. Apicella *et al*., 2016d. Multiaxial fatigue strength to notched specimens made of 40CrMoV13.9. Am. J. Eng. Applied Sci., 9: 1269-1291.

Cao, W., H. Ding, Z. Bin and C. Ziming, 2013. New structural representation and digital-analysis platform for symmetrical parallel mechanisms. Int. J. Adv. Robotic Sys. DOI: 10.5772/56380

Comanescu, A., 2010. Bazele modelarii mecanismelor. E. Politeh. Press, Bucureşti, pp: 274.

Dong, H., N. Giakoumidis, N. Figueroa and N. Mavridis, 2013. Approaching behaviour monitor and vibration indication in developing a General Moving Object Alarm System (GMOAS). Int. J. Adv. Robotic Sys. DOI: 10.5772/56586

Franklin, D.J., 1930. Ingenious mechanisms for designers and inventors. 1st Edn., Industrial Press Inc., New York, pp: ISBN-10: 0831110325, pp: 486.

He, B., Z. Wang, Q. Li, H. Xie and R. Shen, 2013. An analytic method for the kinematics and dynamics of a multiple-backbone continuum robot. IJARS. DOI: 10.5772/54051

Lee, B.J., 2013. Geometrical derivation of differential kinematics to calibrate model parameters of flexible manipulator. Int. J. Adv. Robotic Sys. DOI: 10.5772/55592

Lin, W., B. Li, X. Yang and D. Zhang, 2013. Modelling and control of inverse dynamics for a 5-DOF parallel kinematic polishing machine. Int. J. Adv. Robotic Sys. DOI: 10.5772/54966

Liu, H., W. Zhou, X. Lai and S. Zhu, 2013. An efficient inverse kinematic algorithm for a PUMA560-structured robot manipulator. IJARS. DOI: 10.5772/56403

Mirsayar, M.M., V.A. Joneidi, R.V.V. Petrescu, F.I.T. Petrescu and F. Berto, 2017. Extended MTSN criterion for fracture analysis of soda lime glass. Eng. Fracture Mechanics, 178: 50-59. DOI: 10.1016/j.engfracmech.2017.04.018

Padula, F. and V. Perdereau, 2013. An on-line path planner for industrial manipulators. Int. J. Adv. Robotic Sys. DOI: 10.5772/55063

Perumaal, S. and N. Jawahar, 2013. Automated trajectory planner of industrial robot for pick-and-place task. IJARS. DOI: 10.5772/53940

Petrescu, F. and R. Petrescu, 1995a. Contributions to optimization of the polynomial motion laws of the stick from the internal combustion engine distribution mechanism. ESFA, 1: 249-256.

Petrescu, F. and R. Petrescu, 1995b. Contributions to the synthesis of internal combustion engine distribution mechanisms. ESFA, 1: 257-264.

Petrescu, F. and R. Petrescu, 1997a. Dynamics of cam mechanisms (exemplified on the classic distribution mechanism). SYROM, 3: 353-358.

Petrescu, F. and R. Petrescu, 1997b. Contributions to the synthesis of the distribution mechanisms of internal combustion engines with Cartesian coordinate method. SYROM, 3: 359-364.

Petrescu, F. and R. Petrescu, 1997c. Contributions to maximizing polynomial laws for the active stroke of the distribution mechanism from internal combustion engines. SYROM, 3: 365-370.

Petrescu, F. and R. Petrescu, 2000a. Synthesis of distribution mechanisms by the rectangular (cartesian) coordinate method. University of Craiova.

Petrescu, F. and R. Petrescu, 2000b. The design (synthesis) of cams using the polar coordinate method (the triangle method). Proceedings of the Grafica Conference, University of Craiova, (CUC' 00), Craiova.

Petrescu, F. and R. Petrescu, 2002a. Motion laws for camshafts. Proceedings of the 7th National Symposium with International Participation Computer Assisted Design, (CAD' 02), Braşov, pp: 321-326.

Petrescu, F. and R. Petrescu, 2002b. Camshaft dynamics elements. Proceedings of the 7th National Symposium on International Computer Assisted Design (PAC' 02), Braşov, pp: 327-332.

Petrescu, F. and R. Petrescu, 2003. Some elements regarding the improvement of the engine design. Proceedings of the 8th National Symposium, Descriptive Geometry, Technical Graphics and Design, (GTD '03), Brasov, pp: 353-358.

Petrescu, F. and R. Petrescu, 2005a. The cam design for a better efficiency. Proceedings of the International Conference on Engineering Graphics and Design, (EGD' 05), Bucharest, pp: 245-248.

Petrescu, F. and R. Petrescu, 2005b. Contributions at the dynamics of cams. Proceedings of the 9th IFToMM International Symposium on Theory of Machines and Mechanisms, (TMM' 05), Bucharest, Romania, pp: 123-128.

Petrescu, F. and R. Petrescu, 2005c. Determining the dynamic efficiency of cams. Proceedings of the 9th IFToMM International Symposium on Theory of Machines and Mechanisms, (TMM' 05), Bucharest, Romania, pp: 129-134.

Petrescu, F. and R. Petrescu, 2005d. An original internal combustion engine. Proceedings of the 9th IFToMM International Symposium on Theory of Machines and Mechanisms, (TMM' 05), Bucharest, Romania, pp: 135-140.

Petrescu, F. and R. Petrescu, 2005e. Determining the mechanical efficiency of Otto engine's mechanism. Proceedings of the 9th IFToMM International Symposium on Theory of Machines and Mechanisms, (TMM' 05), Bucharest, Romania, pp: 141-146.

Petrescu, F.I. and R.V. Petrescu, 2011. Mechanical Systems, Serial and Parallel – Course (in romanian). 1st Edn., LULU Publisher, London, UK, pp: 124.

Petrescu, F.I. and R.V. Petrescu, 2012a. Kinematics of the planar quadrilateral mechanism. Engevista, 14: 345-348.

Petrescu, F.I. and R.V. Petrescu, 2012b. Mecatronica-Sisteme Seriale Si Paralele. 1st Edn., Create Space Publisher, USA, pp: 128.

Petrescu, F.I. and R.V. Petrescu, 2013a. Cinematics of the 3R Dyad. Engevista, 15: 118-124.

Petrescu, F.I., Petrescu, R.V., 2013b. Forces and efficiency of cams. Int. Rev. Mechanical Eng.

Petrescu, F.I. and R.V. Petrescu, 2016a. Parallel moving mechanical systems kinematics. ENGEVISTA, 18: 455-491.

Petrescu, F.I. and R.V. Petrescu, 2016b. Direct and inverse kinematics to the anthropomorphic robots. ENGEVISTA, 18: 109-124.

Petrescu, F.I. and R.V. Petrescu, 2016c. Dynamic cinematic to a structure 2R. Revista Geintec-Gestao Inovacao E Tecnologias, 6: 3143-3154.

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado *et al*., 2017a. Yield at thermal engines internal combustion. Am. J. Eng. Applied Sci., 10: 243-251.

Petrescu, R.V., R. Aversa, B. Akash, B. Ronald and J. Corchado *et al*., 2017b. Velocities and accelerations at the 3R mechatronic systems. Am. J. Eng. Applied Sci., 10: 252-263.

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado *et al*., 2017c. Anthropomorphic solid structures n-R kinematics. Am. J. Eng. Applied Sci., 10: 279-291.

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado *et al*., 2017d. Inverse kinematics at the anthropomorphic robots, by a trigonometric method. Am. J. Eng. Applied Sci., 10: 394-411.

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado *et al*., 2017e. Forces at internal combustion engines. Am. J. Eng. Applied Sci., 10: 382-393.

Petrescu, RV., R. Aversa, B. Akash, R. Bucinell and J. Corchado *et al*., 2017f. Gears-part I. Am. J Eng. Applied Sci., 10: 457-472.

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado *et al*., 2017g. Gears-Part II. Am. J. Eng. Applied Sci., 10: 473-483.

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado *et al*., 2017h. Cam-gears forces, velocities, powers and efficiency. Am. J. Eng. Applied Sci., 10: 491-505.

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado *et al*., 2017i. Dynamics of mechanisms with cams illustrated in the classical distribution. Am. J. Eng. Applied Sci., 10: 551-567.

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado *et al*., 2017j. Testing by non-destructive control. Am. J. Eng. Applied Sci., 10: 568-583.

Petrescu, R.V., R. Aversa, A. Apicella and F.I.T. Petrescu, 2017k. Transportation engineering. Am. J. Eng. Applied Sci., 10: 685-702.

Petrescu, R.V., R. Aversa, S. Kozaitis, A. Apicella and F.I.T. Petrescu, 2017l. The quality of transport and environmental protection, part I. Am. J. Eng. Applied Sci., 10: 738-755.

Petrescu, R.V., R. Aversa, A. Apicella and F.I. Petrescu, 2016. Future medicine services robotics. Am. J. Eng. Applied Sci., 9: 1062-1087.

Petrescu, F.I., B. Grecu, A. Comanescu and R.V. Petrescu, 2009. Some mechanical design elements. Proceeding of the International Conference on Computational Mechanics and Virtual Engineering, (COMEC' 2009), Braşov, pp: 520-525.

Petrescu, F.I., 2011. Teoria Mecanismelor si a Masinilor. 1st Edn., Create Space Publisher, USA, pp: 432.

Petrescu, F.I.T., 2015a. Geometrical synthesis of the distribution mechanisms. Am. J. Eng. Applied Sci., 8: 63-81.

Petrescu, F.I.T., 2015b. Machine motion equations at the internal combustion heat engines. Am. J. Eng. Applied Sci., 8: 127-137.