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Modified Hill Cipher with Key Dependent Permutation and Circular Rotation

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Abstract: In this study, we have modified the Hill cipher, by including a permutation and circular rotation into the cipher. Here both the permutation and the rotation depend upon the key. From the cryptanalysis and the avalanche effect, discussed in this study, we notice that the strength of the cipher is significant.

Key words: Modular arithmetic inverse, key dependent permutation, key dependent rotation

INTRODUCTION

The study of the block ciphers[1] which depends upon a symmetric key attracted the attention of researchers, in the first part of last century. Hill developed a cipher, by taking a key in the form of a matrix and using the concept of modular arithmetic inverse. However, it was established that the cipher can be broken by the known plaintext attack.

In a recent research[2], we have examined the cryptology of the Hill cipher and introduced a systematic procedure for the development of the modular arithmetic inverse of a matrix. In this we have shown that the modular arithmetic inverse of a matrix exists only when the matrix is non-singular, and the value of the determinant is relatively prime to N, where N is a positive integer with which modulo operation is carried out. In this we have noticed that the Hill cipher is vulnerable as the matrix multiplication and the modular arithmetic lead to a direct mapping, without any transposition, connecting the plaintext and the ciphertext.

In the present research our objective is to develop a cipher which involves different types of transformations such as permutation and circular rotation in addition to matrix multiplication and modular arithmetic. In this investigation we have made use of iteration in order to enhance the strength of the cipher. This cipher is expected to be a very strong one, and it cannot be broken by any cryptanalytic attack.

DEVELOPMENT OF THE CIPHER

Consider a plaintext P. Let us suppose that it can be represented in the form of a matrix given by

$$P = [Pij],\ i = 1 \text{ to } n,\ j = 1 \text{ ton} \qquad (1)$$

Consider the key matrix K where

$$K = [Kij],\ i = 1 \text{ to } n,\ j = 1 \text{ to } n \qquad (2)$$

Let us suppose that each element in the key matrix lies between 0 and 127.
Let

$$C = [Cij]\ i = 1 \text{ to } n,\ j = 1 \text{ to } n \qquad (3)$$

be the ciphertext which can be obtained from the plaintext by applying a set of transformations.

In order to facilitate our analysis, let us represent the above matrices in terms of their corresponding vectors p,k,c given by

$$p = [p_1,p_2,p_3\ldots p_{n^2}],$$

$$k = [k_1,k_2,k_3\ldots k_{n^2}]$$

and

$$c = [c_1,c_2,c_3\ldots c_{n^2}].$$

In writing these vectors we have arranged the elements of the corresponding matrices in a row wise manner.

The procedures of encryption and decryption concerned to this cipher are shown inFig.1.

This cipher, depends upon the functions: (1)Convert( ), (2)Iconvert( ), (3)Permute( ), (4) Ipermute ( ), (5)Lrotate( ), and (6)Rrotate( ).

The function Permute( ) permutes the binary bits of a plaintext vector in accordance with the numbers present in the key vector. This process of permutation can be described as follows:

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Convert the components of p, namely, p_1, $p_{2,}$ p_3….p_n2 into their corresponding binary bits. Thus we get the binary bits in the form

$$p_{11}\ p_{12}\ ,\&\ \ p_{17}\ ,\ p_{21}\ p_{22}\ \ \&\ \&. \qquad (4)$$
$$p_{27}\ \ \&\ ..\ p_{n\,21}\ p_{n\,22}\ \ \&\ ..\ p_{n\,27}$$

where p_{11} $p_{12,}$ … p_{17} are the binary bits corresponding to p_1, p_{21} p $_{22,}$……. p_{27} are the binary bits corresponding to p_2, and p_{n^21} p $_{n^22}$…..p $_{n^27}$ are the binary bits corresponding to $p_n{}^2$.

Let the key vector (obtained from the key matrix) be

$$k_1, k_2, ...k_n^2 \qquad (5)$$

As the numbers in the plaintext are lying between 0 and 127 we have only 7 bits for every number.

Here we have $7n^2$ binary bits in the plaintext and n^2 numbers in the key. Thus we divide the string of $7n^2$ binary bits into 7 substrings wherein each substring contains n^2 binary bits. Then we interchange the first bit of the first substring with the k_1th bit of the entire string. Similarly we interchange the second bit of the first substring with the k_2th bit of the entire string. We continue this process until all the bits in the first substring are over. In the same way, we carry out the process with the second substring and the third substring, and so on. However, if any one of the numbers in k_1 to k_n2 exceeds $7n^2$, the length of the string, then the interchange is ruled out, and it will not be done. For a detailed discussion of the permutation, let us refer to the example given in Appendix 1.

The function Convert( ) is developed for converting a vector into a two dimensional matrix. Iconvert( ) performs the reverse process to that of Convert( ).

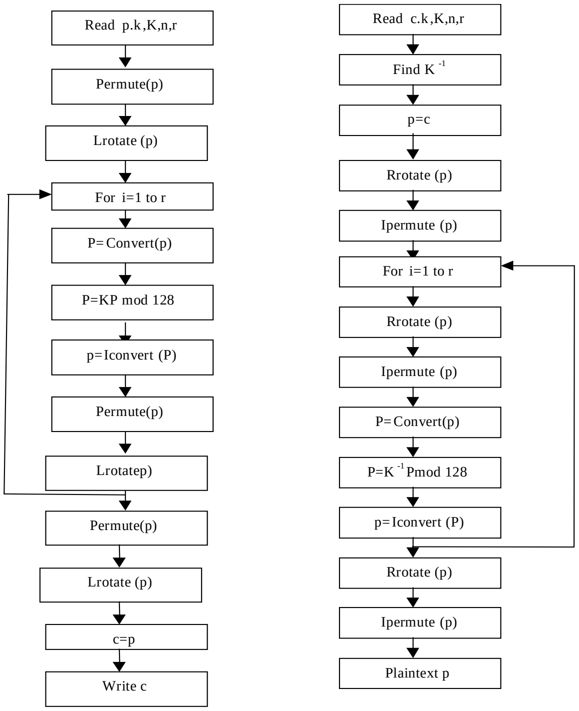
Now, the process of left circular rotation denoted by Lrotate( ) and the process of right circular rotation denoted by Rrotate( ) are explained below. As it is mentioned above let us represent the numbers p_1, $p_{2,}$ p_3 …. $p_n{}^2$(the components of p) in terms of their binary bits. Let the string of binary bits be written as

$$p_{11}p_{12},\&p_{17}, p_{21}p_{22},.\&\&.p_{27}, p_{n21}\ \&\&.p_{n27} \qquad (6)$$

The key vector k has the components k1, k_2…... $k_n{}^2$, where each one of these numbers lies between 0 and 127.Let us now obtain a set of numbers d_i, such that

$$d_i = k_i \bmod n_2,\ i = 1\, to\, n_2 \qquad (7)$$

Now we rotate the set of binary bits given by (6) through d_i positions to the left. The function Rrotate( ) is exactly opposite to that of Lrotate( ).



(a) process of encryption (b) process of decryption

Fig. 1: Schematic diagram of the cipher

In Fig.1 r denotes the number of iterations and it is taken as 16 in this analysis. K^{-1} is the modular arithmetic inverse of K.

In what follows we design the algorithms for encryption and decryption of the cipher.

ALGORITHMS

Algorithm for encryption:

```
{
1   Read n,k,K,p,r;
2   Permute(p);
3   Lrotate(p);
4   for i = 1 to r
    {
5   P = Convert(p);
6   P = KP mod 128;
7   p = Iconvert(P);
8   Permute(p);
9   Lrotate(p);
    }
10  Permute(p);
```

```
    Lrotate(p);
    c = p;
    write c;
}
```

Algorithm for decryption:

```
{
    Read n,k,K,c,r;
    Find $K^{-1}$;
    p=c;
    Rrotate(p);
    Ipermute(p);
    for i= 1 to r
    {
        Rrotate(p);
        Ipermute(p);
        P=Convert(p);
        P= $K^{-1}$ P mod 128;
        p=Iconvert(P);
    }
    Rrotate(p);
    Ipermure(p);
    write p;
}
```

Algorithm for modular arithmetic inverse: // A is an nxn matrix. N is a positive integer with which modular arithmetic inverse is carried out. Here N = 128.

```
{
    1. Find the determinant of A. Let it be denoted by $\Delta$, where $\Delta \neq 0$.
    2. Find the inverse of A. The inverse is given by $[A_{ji}]/\Delta$.
    3. for i = 1 to N
    {
    if ( (i$\Delta$) mod N = 1 )  d = i;
            //$\Delta$ is relatively prime to N.
    break;
    }
B=( d[$A_{ji}$] ) mod N
    // B is the modular arithmetic inverse of A
}
```

ILLUSTRATION OF THE CIPHER

Consider the plaintext given below: In order to do a good business in any other country, we must have cordial relationship with all other localities of the country. We must supply only those commodities for which very good demand is there.

Let us focus our attention on the first 16 characters given by

In␢order␢to␢do␢a.

This plaintext can be written in the form

$$p = [73\ 110\ \ 32\ \ 111\ \ \ 114\ \ 100\ \ 101\ \ 114 \quad 32\ \ 116\ \ 111\ \ 32\ \ 100\ \ 111\ \ 32\ \ 97] \tag{8}$$

Let us choose the key vector in the form

$$k = [18\ \ 4\ \ 7\ \ \ 3\ \ \ 4\ \ \ 6\ \ \ 5\ \ \ 42\ \ \ 40 \quad 31\ \ \ 9\ \ 22\ \ \ 35\ \ \ 17\ \ \ 23\ \ \ \ 71\ ] \tag{9}$$

On using (4.1), (4.2) and the encryption algorithm 3.1, we get the ciphertext c in the form

$$c = [114\ \ 114\ \ 102\ \ 123\ \ 67\ \ \ 61\ \ 85\ \ 34 \quad 93\ \ 106\ \ \ 11\ \ 56\ \ \ 122\ 92\ \ \ 71\ \ 9] \tag{10}$$

The inverse of the key matrix K can be obtained as

$$K^{-1} = \begin{bmatrix} 75 & 85 & 127 & 37 \\ 120 & 115 & 69 & 84 \\ 110 & 101 & 22 & 64 \\ 115 & 125 & 76 & 114 \end{bmatrix} \tag{11}$$

On using the decryption algorithm given in 3.2, we get back the plaintext in the form p = [73 110 32 111 114 100 101 114 32 116 111 32 100 111 32 97]

This is same as (8).

CRYPTANALYSIS

Let us now consider the cryptanalysis of the cipher. In this the length of the key is $7n^2$ binary bits. Thus the size of the key space is 2^{7n^2}. Hence the cipher cannot be broken by ciphertext only attack, when n> = 4.

In the case of the known plaintext attack, though we know as many pairs of plaintext and ciphertext as we like, here it is not possible to form a direct equation connecting the plaintext and the ciphertext matrices as there are several transformations, namely, permutation and rotation involved in the process of encryption. Thus the cipher cannot be broken by the known plaintext attack also. Further it may be mentioned that no special choice of the plaintext/ciphertext will enable the cryptanalyst to break the cipher.

In the light of the above discussion we find that this cipher is a very strong one and it cannot be broken by any cryptanalytic attack.

AVALANCHE EFFECT

Let us consider the avalanche effect. On changing the first character ‘I’ in the plaintext (8) to ‘H’, the plaintext in its binary form changes by one bit. On using the encryption algorithm, on the modified plaintext, we get the ciphertext in the form,

```
00000000101011001111011100110
11101101101100100000001111100 11      (12)
00010111110100101010010011110 0
```

On converting (10) into its binary form, we get

```
11100101110010110011011111011
10000110111110110101010110011
10111011101010000101101110 00      (13)
11110101100110100011100010 01
```

On comparing (12) and (13) we find that the two ciphertexts differ in 52 bits.

Now let us change the key by one bit. To this end, we replace $k_2 = 4$ by $k_2 = 5$.Then on using the plaintext (8), the modified key, and the algorithm, we get the ciphertext in the form

```
01111000010100000001011011011
00111011111101001111100111 10
10001110110011000000110100 10      (14)
00001001110011000111001110 01
```

On comparing the ciphertexts (12) and (14) we notice that these two ciphertexts differ in 55 bits.

From the above discussion, it is clearly seen that the cipher exhibits a strong avalanche effect.

COMPUTATIONS AND CONCLUSIONS

In this paper we have developed a block cipher by introducing a key dependent permutation and a key dependent circular rotation. The algorithms developed in this analysis are implemented in C language.

The ciphertext corresponding to the entire plaintext is obtained in hexadecimal notation as follows.

```
72 72 66 7B 43 3D 55 22 5D 6A 0B 38 7A 5C 47 09
3A 60 77 3A 7B 22 52 7A 35 0C 3F 3B 50 71 53 23 15
4A 21 12 7C75 1A 53 33 1D 13 0C 54 3B OE 20 0D 75
65 34  2C 09 7B 3E 17 30 76 3F 59 23 45  65 3E  61 75
4F 4F 51  06  00 38  28 45  6B 18  15 74 1E 06 52 57
03  7E  31  52  7F  16  70  6C  3A  6D  67  61  6B  66
20  05  2E  7A  70  75  08  3D  15  45  07   6C  21 72 15
1C  09   71  05  13  4E  19   50   2F  04   71  6C  5E  30
5D  4A  52  41  62  69  62  5E  61   6D   48  4A  48  4A
2E  0C  7E  55  35 0D  06  3C  4E  38   7B  33 4A  69
48  36  0B  32  26  5E  2F  12   6E   39  0A 5D  0F   30
2C  31  7A   7A  1D  03   40  63  14  49 74  68  54   31
03   75   4F  07  2D   24   22   62  2A  27  17  30   30 13
```

From the above analysis we conclude that the strength of the cipher increases enormously as the permutation and the rotation are key dependent, and they include a lot of confusion into the structure of the plaintext at various stages of the iteration.

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