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Cut-Set Based Method to Determine the Maximum Demand Accommodated by a Multi-State Network

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Abstract: The reliability of a multi-state network is defined as the probability that the network can successfully send d (demand) units of data from the source to the sink. To predict the value of maximum demand (d_{max}) that could be accommodated by a network, a cut-set based approach is presented in this study. The presented approach is simple and easy to implement. The proposed method was applied to many examples studied in literature to illustrate its efficiency. The results show that the reliability value at maximum demand ($R_{d_{max}}$) is less than any R_d.

Keywords: Cut-Set, Maximum Demand, System Reliability, Multi-State Network

Introduction

In the case of existing multi-state components with a limited number of different states, each state has a different capacity and probability, the network is considered as a multi-state network. Given the demand (d), the reliability (R_d) is defined as the probability of the network capacity level greater than or equal to d. Various algorithms were presented to evaluate R_d, (Lin *et al.*, 1995; Lin, 2001a). All of these methods assumed that all minimal paths (Chen and Lin, 2012; Yeh, 2016) to be known in advance. Other methods are presented to improve searching the *d-MPS* with knowing MPS in advance or without (Yeh, 1998; 2002; 2018; Lin, 2001a; Bai *et al.*, 2015; Chen, 2014; Chen and Lin, 2016; Lin and Chen, 2017; 2019; Xu *et al.*, 2019). Also, many algorithms presented to evaluate R_d in terms of Minimal Cuts (MCS) vectors to a given demand d, *d-MCS* based, (Jane *et al.*, 1993; Lin, 2001b; 2003a; Jane and Laih, 2010; Yeh *et al.*, 2015). The idea was to find all *d-MCS* prior to calculating network reliability between the source and the sink nodes. The condition is that all MCS (Abel and Bicker, 1982) are known in advance. In addition, some researchers presented methods to search the *d-MCS*, (Yeh, 2005; 2008; Chaturvedi and Mishra, 2009; Forghani-Elahabad and Mahdavi-Amiri, 2014).

The enumeration of *d*-MPS was considered as an NP-hard problem and developing an efficient algorithm that depends on the network maximum flow to enumerate all *d*-MPS without prior knowledge of MPS.

Therefore, evaluating the system reliability of multi-state network using *d-MPS* or *d-MCS* depends on the demand value. Consequently, this paper focuses on determining the maximum demand accommodated by a multi-state network to save the effort in searching *d-MPS* or *d-MCS*. In addition, it helps the decision-maker or network administrator to accept or refuse the required demand. Furthermore, it helps the designer and researcher to manipulate the problem of transmitting the maximum demand over the network to increase its performance. This paper presents an algorithm based on the Cut-set of both the source and the sink nodes to determine the maximum demand.

This paper is structured as follows. Section 2 presents notations and assumptions. Section 3 presents preliminaries to evaluate R_d. Section 4 describes the proposed algorithm. Section 5 provides illustrative examples and studied cases. Section 6 offers our conclusions.

Reliability Evaluation Algorithm

The reliability of a stochastic flow network R_d under the demand d is evaluated in terms of d-MP based on the following:

1. Deduce the flow vector $F = (f_1,f_2,\ldots,f_{np})$ according to (Lin *et al.*, 1995; Lin, 2001b), that satisfies:

$$\sum_{j=1}^{np}\left\{ f_j | a_i \in mp_j \right\} \le M^i, i = 1,2,\ldots,m \quad (1)$$

$$\sum_{j=1}^{np} f_j = d \quad (2)$$

$$f_j \leq L_j \tag{3}$$

2. Generate the capacity vector $X = (x_1,x_2,\ldots,x_m)$ from $F = (f_1,f_2,\ldots,f_{np})$ by using the following equation:

$$x_i = \sum_{j=1}^{np} \left\{ f_j | a_i \in mp_j \right\}, i = 1,2,\ldots,m \tag{4}$$

3. Assume that the generated lower boundary vectors are $X^1, X^2,\ldots,X^q$, then R_d is given by:

$$R_d = \Pr\left\{ \bigcup_{i=1}^{q} \left\{ X | X \geq X^i \right\} \right\} \tag{5}$$

Is evaluated by inclusion-exclusion or RSDP (Zuo *et al*., 2007) used here.

Algorithm Based on Cut-Sets to Determine d_{max}

Begin

STEP 1. Detect the source and sink nodes

STEP 2. Determine the cut-set for both the source and the sink nodes as:

$$mc(s) = \{a_e \mid a_e \text{ connects the source node } s\}$$
$$mc(t) = \{a_e \mid a_e \text{ connects the destination node } t\}$$

STEP 3. Find the sum of the maximum capacity for mc(s) and mc(t) as:

$$\mu_s = \sum_e M_e | a_e \in mc(s) \text{ and}$$
$$\mu_t = \sum_e M_e | a_e \in mc(t)$$

STEP 4. Determine the value of d_{max} as:

$$d_{\max} = Minimum(\mu_s, \mu_t) + \varepsilon$$

Where, ε is an integer and $0 \leq \varepsilon \leq |\mu_s - \mu_t|$

STEP 5. If $\varepsilon = 0$, then set d_{max} = Minimum (μ_s, μ_t) and evaluate $R_{d_{\max}}$ as:

descriped in section 2. otherwise go to Step 6.

STEP 6. For $\varepsilon = |\mu_s - \mu_t|$ down to 0 do

STEP 6.1. Set d_{max} = Minimum (μ_s, μ_t) + ε

STEP 6.2. Check if there is at least one solution using section 2, then print out d_{max} and $R_{d_{\max}}$ and go to End.

STEP 6.3. End do

STEP 6.4. Print out d_{max} and $R_{d_{\max}}$ and go to End.

STEP 6.5. End do

End

Illustrative Examples

Four Nodes Network

Consider the following network given in Fig. 1 with link capacities and probabilities are shown in Table 1. This network with the given information studied in (Lin *et al*., 1995; Lin, 2001b; Yeh, 2018; Yeh, 2005; Zuo *et al*., 2007; Yeh, 2010; Niu and Xu, 2012).

The Following Steps Show How to use the Proposed Algorithm to Determine d_{max}

Begin

STEP 1. The source and sink nodes are 1 and 4 respectively.

STEP 2. Determine the cut-set for 1 and 4 are:

$$mc(1) = \{a_1, a_5\} \text{ and } mc(4) = \{a_2, a_6\}$$

STEP 3. Calculate μ_1 and μ_4:

$$\mu_1 = M_1 + M_5 = 4 \text{ and } \mu_4 = M_2 + M_6 = 4$$

STEP 4. Determine the value of $d_{\max}$ as:

$$d_{\max} = Minimum(\mu_1, \mu_4)$$
$$= Minimum\ (4, 4) = 4 + \varepsilon$$

STEP 5. Because $\varepsilon = |\mu_1 - \mu_4| = |4-4| = 0$, then $d_{\max} = 4$.

End

Then, the maximum demand accommodated by this network is 4.

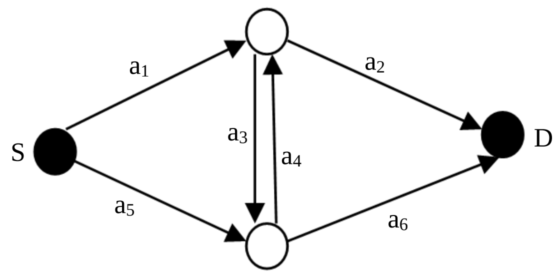

Fig. 1: Four nodes network

Table 1: The Arcs' information

Arc	Capacities				Probabilities			
a_1	0	1	2	3	0.05	0.10	0.25	0.60
a_2	0	1	2	-	0.10	0.20	0.70	-
a_3-a_4	0	1	-	-	0.10	0.90	-	-
a_5	0	1	-	-	0.20	0.80	-	-
a_6	0	1	2	-	0.10	0.20	0.70	-

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Four Nodes Network with Ten Components

In the case of a node failure, the network given in Fig. 2 with the information is shown in Table 2 studied in (Lin, 2001a).

Applying the Proposed Algorithm to Determine d_{max}

STEP 1. The source and sink nodes are 1 and 4 respectively.
STEP 2. Determine the cut-set for 1 and 4 are:

$$mc(1) = \{a_1, a_5\} \text{ and } mc(4) = \{a_2, a_6\}$$

STEP 3. Calculate μ_1 and μ_4*:*

$$\mu_1 = M_1 + M_5 = 5 \text{ and } \mu_4 = M_2 + M_6 = 6$$

STEP 4. Determine the value of d_{max} *as:*

$$d_{max} = Minimum\ (\mu_1, \mu_4) = Minimum\ (5, 6) + \varepsilon = 5 + \varepsilon$$

STEP 5. Because $\varepsilon = |\mu_1 - \mu_4| = |5-6| = 1$, then *go to Step 6.*
STEP 6. For $\varepsilon = 1$ *down to* 0 *do*
STEP 6.1. $\varepsilon = 1$ *then* $d_{max} = 5+1 = 6$.
STEP 6.2. Using section 2, no solution found for $d_{max} = 6$.
STEP 6.3. $\varepsilon = 0$ *then* $d_{max} = 5$ *and* $R_5 = 0.824242$. *Then End the algorithm.*
STEP 6.4. End do
End

Five Nodes Network

This section presents another five nodes network with eight links, (Lin, 2003b), as shown in Fig. 3 and the components information are given in Table 3.

Begin
STEP 1. The source and sink nodes are 1 and 4 respectively.
STEP 2. Determine the cut-set for 1 and 4 are:

$$mc(1) = \{a_1, a_3\} \text{ and } mc(5) = \{a_4, a_6, a_8\}$$

STEP 3. Calculate μ_1 and μ_4*:*

$$\mu_1 = M_1 + M_3 = 5 \text{ and } \mu_5 = M_4 + M_6 + M_8 = 8$$

STEP 4. Determine the value of d_{max} *as:*

$$d_{max} = Minimum\ (\mu_1, \mu_5) + \varepsilon = Minimum\ (5, 8) = 5 + \varepsilon$$

STEP 5. Because $\varepsilon = |\mu_1 - \mu_4| = |5-8| = 3$, then *go to Step 6.*
STEP 6. For $\varepsilon = 3$ down to 0 *do*
STEP 6.1. $\varepsilon = 3$ *then* $d_{max} = 5+3 = 8$.
STEP 6.2. Using section 2, no solution found for $d_{max} = 8$.
STEP 6.3. $\varepsilon = 2$ *then* $d_{max} = 5+2 = 7$.
STEP 6.4. Using section 2, no solution found for $d_{max} = 7$.
STEP 6.5. $\varepsilon = 1$ *then* $d_{max} = 5+1 = 6$.
STEP 6.6. Using section 2, no solution found for $d_{max} = 6$.
STEP 6.7. $\varepsilon = 0$ *then* $d_{max} = 5+0 = 5$.
STEP 6.8. Using section 2, we found $d_{max} = 5$ *and* $R_5 = 0.572599$. *Then go to End.*
STEP 7. End do
End

Table 2: The Arcs' information

Arc	Capacity	Probability	Arc	Capacity	Probability
a_1	2	0.980	a_4	3	0.960
	1	0.010		2	0.020
	0	0.010		1	0.010
a_2	3	0.960		0	0.010
	2	0.020	a_5	3	0.970
	1	0.010		2	0.010
	0	0.010		1	0.010
a_3	2	0.980		0	0.010
	1	0.010	a_6	3	0.960
	0	0.010		2	0.020
a_7	6	0.955		1	0.010
	5	0.005		0	0.010
	4	0.005	a_9	4	0.966
	3	0.005		3	0.050
	2	0.010		2	0.050
	1	0.010		1	0.010
	0	0.010		0	0.010
a_8	5	0.965	a_{10}	5	0.965
	4	0.005		4	0.005
	3	0.005		3	0.005
	2	0.005		2	0.005
	1	0.010		1	0.010
	0	0.010		0	0.010

Table 3: The Arcs' information

Arc	Capacity	Probability	Arc	Capacity	Probability
a_1	3	0.80	a_6	4	0.60
	2	0.10		3	0.20
	1	0.05		2	0.10
	0	0.05		1	0.05
a_2	3	0.80		0	0.05
	2	0.10	a_7	5	0.55
	1	0.05		4	0.10
	0	0.05		3	0.10
a_3	2	0.85		2	0.10
	1	0.10		1	0.10
	0	0.05		0	0.05
a_4	1	0.90	a_8	3	0.80
	0	0.10		2	0.10
a_5	1	0.90		1	0.05
	0	0.10		0	0.05

Table 4: More studied cases

No.	*ncp*	*np*	ε	$d_{\max}$	$R_{d_{\max}}$	Studied by
1	5	3	0	4	0.2041200	Lin *et al*. (1995; Lin, 2001a)
5	14	7	0	10	0.5685590	Lin (2004)
6*	21	13	0	5	0.9481130	Jane and Laih (2008)
7	30	44	0	3	0.1110566	
3	5	3	0	18	0.7002650	Hassan (2016)
4	6	4	0	11	0.1118240	Chen and Lin (2016)
8	16	9	3	7	0.8338490	Xu *et al*. (2019)

*The reliability values are different from those obtained by (Jane and Laih, 2008), we verified and asserted that our values are the correct ones after contacting the authors

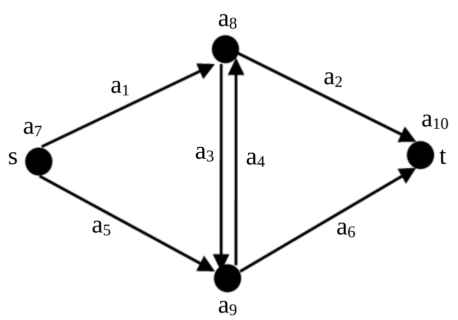

Fig. 2: Four nodes Network with ten components

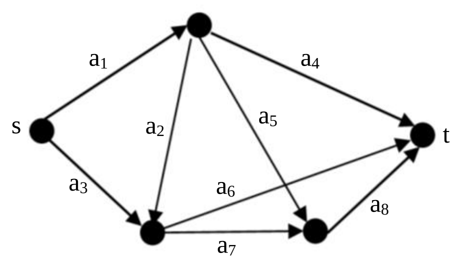

Fig. 3: Five nodes network with ten components

More studied Cases

This section presents additional examples taken from literature as shown in Table 4.

Conclusion

The paper studied how to calculate the maximum value of the demand ($d_{\max}$) that can be accommodated by a flow network. A simple algorithm based on cut sets is presented to find $d_{\max}$. In some cases, $d_{\max}$ is determined exactly and directly when there is no difference between the sum of maximum states of source and sink cut-sets (μ_s-μ_t) i.e., $\varepsilon = 0$. Otherwise, ε ranges from 0 to the difference between the two sums ($|\mu_s-\mu_t|$), in this case, the value of $d_{\max}$ lies inside an interval, Minimum (μ_s-μ_t) $\leq d_{\max} \leq$ Minimum (μ_s-μ_t) + ε.

Also, we got an important conclusion that $R_{d_{\max}} < R_d$, for all $d_{\max}>d$. Finally, this study helps the network administrator or decision-maker previously decide to accept the demand or refuse.

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Author's Contributions

All authors are equally contributed in this work and the article.

Ethics

Authors confirm that this manuscript has not been published elsewhere and that no ethical issues are involved.

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Notations

n	Number of nodes.
m	Number of arcs (links).
ncp	Number of components ($n + m$ or m Only)
np	Number of paths.
MC	Minimal cuts
$mc(i)$	Minimal cut set for node i
$mc(s)$	Minimal cut set for source node s
$mc(t)$	Minimal cut set for destination node t

M	$M^1, M^2,\ldots,M^m$), M^e is the maximum capacity of a component a_e.
$d_{\max}$	The maximum demand accommodated by the network.
μ_s	The maximum capacity of a source cut-set.
μ_t	The maximum capacity of a destination cut-set.
MPs	Minimal paths.
mp_j	Minimal path no. j; $j = 1, 2, \ldots, m$.
L_j	The maximum capacity of mp_j; $L_j = \min\{M^i \mid a_i \in mp_j\}$.
R_d	The reliability of a multi-state network under the demand d.