

## **Backscattered Acoustic Energy by Monobubbles Experimental Approach and Statistical Study of the Attenuation**

<sup>1</sup>Ali Khelil and <sup>2</sup>Claude Mante

<sup>1</sup>Department of Physics, Mentouri University, Ain-El-Bey Road,  
25000 Constantine, Algeria

<sup>2</sup>Oceanological Center of Marseilles,  
Campus of Luminy, Case 901, F13288 Marseilles, Cedex 9, France

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**Abstract: Problem statement:** As the number of air bubbles in the sea is very high, they are so many acoustic diffusers who make illegible the recordings the purpose of which is to quantify the alive bodies. The signals backscattered by air bubbles constitute a parasite in offshore recordings and must be eliminated. It is planned to finalize techniques allowing the localization and the identification of a signal backscattered by an air bubble. Once this type of signal was localized and identified on an offshore recording, it is easy to eliminate it. From then, we could have recordings where the only diffusers would be alive bodies like the zooplankton. **Approach:** We began a work of characterization of signals of bubbles to discriminate between them and those backscattered by alive diffusers. We realized in laboratory a bench test then we finalized an original method of production of air bubbles with known size in a liquid medium. Five types of monobubbles were generated in a water column by a technique using a peristaltic pump. This technique allowed obtaining a continuous water flow carrying same-sized air bubble. The bubbles radii were calculated from the measure of rise limit speed. The acoustic responses of these bubbles (to a frequent wide bandwidth ultrasonic wave) were studied by statistical methods in order to determine the variation of the energy backscattered by a calibrated bubble according to its depth. **Results:** Besides the production technique of calibrated bubbles that was finalized, we established that the variation of backscattered energy according to depth can be explained by simple exponential models which permitted to estimate the constant of absorption. **Conclusion:** The coming step will be to correct the signal of the effect of the absorption of energy by the middle, then to elaborate a protocol of localization of the signals of bubbles on recordings where multiple diffusers appear. The results had to be refined and adapted for in-situ applications.

**Key words:** Acoustics, air bubbles, absorption, model, variance analysis

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### **INTRODUCTION**

Air bubbles, or more generally gas bubbles, exist in the sea ; most of them are situated just below the free surface. Generally, small-sized ones (a few dozens microns) are invisible and they eliminate by dissolution<sup>[1]</sup>. The bigger ones disappear by rising up to the surface (floatability)<sup>[2]</sup>. Their origin may be mechanical<sup>[3,4]</sup>, biological<sup>[5]</sup> or other<sup>[1]</sup>. The knowledge of the characteristics of gas bubbles in natural medium has become important about various subjects as ocean-atmosphere exchanges<sup>[3]</sup>, low atmosphere visibility, clouds development, surface chemistry, vertical transportation, underwater acoustics and its military applications<sup>[6]</sup> and carbon cycle research.

The study proposed in this article constitutes the first part of a research subject developed by the team

"Acoustic of Particles in the Sea" of the "Oceanological Center of Marseilles". As the number of air bubbles, especially those close to the sea surface, is very high<sup>[2,7,8]</sup>, they are so many acoustic diffusers who make illegible the recordings the purpose of which is to quantify the alive bodies. In this area of research, it is planned to finalize techniques allowing the localization and the identification of a signal backscattered by an air bubble. Once this type of signal was localized and identified on an offshore recording, it is easy to eliminate it. From then, we could have recordings where the only diffusers would be alive bodies like the zooplankton<sup>[5]</sup>.

Because of the difficulties of offshore experiment, the choice was to begin in laboratory; thus to construct a pilot study and make a diphasic environment: Water-air bubbles.

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**Corresponding Author:** Ali Khelil, Dpartment of Physics, Mentouri University, Ain El Bey Road, 25000 Constantine, Algeria

For this research, air bubbles have been studied from the surface of a water column, outside their resonance domain<sup>[9,10]</sup>. However, the acoustic energy backscattered by bubbles outside resonance depends on their immersion depth. A study about the variation of this energy according to depth allows to correct the signal from depth effect and to determine the true response of a bubble. A statistical study of the absorption of acoustic energy backscattered by calibrated Monobubbles has been started and proposed<sup>[10,11]</sup>. In a first part, we describe an original method of air bubbles production in a water column. Then, we evoke techniques used to record acoustic answers (backscattered energy) of these bubbles to an ultrasound wave.

At last, by variance and covariance analyses<sup>[12,13]</sup>, a model is proposed in order to explain energy variation backscattered by bubbles according to their depth.

### MATERIALS AND METHODS

**Description of the experimental training:** In experimental acoustic studies on gas bubbles in liquid medium, two ways are possible to quantify the bubble effect on the acoustic wave propagation. The first one uses two transducers, an emitter and a receiver, which limits the investigation range to the interval between this two transducers<sup>[14]</sup>. The second way uses one transducer as a vertical sonder with only one transducer which works alternatively as an emitter, then a receiver<sup>[5]</sup>. This last method has been adopted for this research because it allows to study also marine organisms concentrations in the deepest layers of the sea.

The experimental training (Fig. 1) is made up of a cylindrical Plexiglas tube (water column), a peristaltic pump to draw up air and water, a signal generator, a broad band transducer of 500 kHz central frequency and a Nicolet (490) numerical oscilloscope which allows to acquire signals at high speed.

**Bubbles production:** Water and air are pumped simultaneously into calibrated pipes, which provides a constant water flow carrying air bubbles. This flow flows down towards the bottom of the water column, where then a bubbles string rise is obtained. So, the generated bubbles are considered to be of the same size when arriving in the water column. To modify the bubble size, it is necessary to change the calibrated pipe that draws up the air, each different pipe providing different size bubbles type. The pump flow being slow, the number of liberated bubbles is also very low. Bubbles rise, one by one, along an almost rectilinear trajectory in the water column.

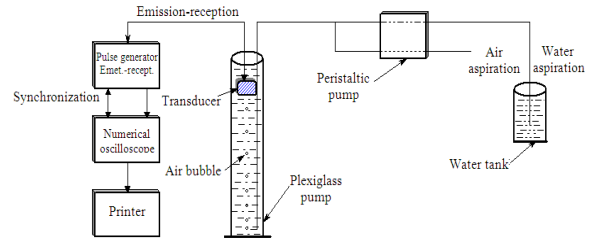


Fig. 1: Diagram of the training system

Table 1: Some statistics on path durations of five types of monobubbles, measured at 1m distance

	Type 1	Type 2	Type 3	Type 4	Type5
Air conduit type $\phi_c$ (mm)	0.005	0.0075	0.010	0.020	0.030
Average $t_{moy}$ (s)	11.64	11.12	10.26	8.77	7.12
Median $t_m$ (s)	11.67	11.06	10.28	8.66	6.97
Variance $\sigma_t$ (s)	0.97	0.12	0.07	0.56	0.32
Standard deviation $s_t$ (s)	0.98	0.35	0.26	0.75	0.57
Deviation from average $\Delta t_{moy}$ (s)	0.70	0.26	0.20	0.56	0.47
Deviation from median $\Delta t_m$ (s)	0.38	0.22	0.16	0.48	0.44

Averages  $t_{moy}$  and medians  $t_m$  decrease linearly with diameter of air pipe  $\phi_c$ . The variances  $\sigma_t$  are very different from one to another. The standard deviation  $s_t$  presents no orderly relationship with bubble sizes

**Rise speed measure and average radii:** The obtained bubbles must be small to be quite spherical (if it is not the case, the air pipe must be changed, decreasing air pipe diameter  $\phi_c$ ). As well, the movement must be rectilinear and unvarying (avoiding the case where bubbles could overtake others). The distance covered by the bubbles between two selected marks is timed. The lower mark must be slightly distant from the bubble exit point in order that the movement should be quite uniform (to avoid the influence of a possible initial speed).

Around thirty bubbles are timed on the chosen way (1 m) and as the bubbles are same-sized, the average rise speed is calculated using the average time<sup>[11,15]</sup>. This experiment is repeated several times to be insured that a chosen air-pipe always give same-sized bubbles. The way time of the bubbles and the statistical calculus are presented in Table 1.

These results show that:

- The values of variances  $\sigma_t$  are very different from one to another and do not have any connection with the type of the pipe. This means that the measures are totally distinct, that is to say that the background noise characteristics are different from one measure to another
- The values of average  $t_{moy}$  and the median  $t_m$  being practically equal in a same serie of five types of measures, there is no aberrant value and the errors distribution is symmetrical

Table 2: Average radii of the five types of produced bubbles, deduced from rise limit speed

Air conduit type $\Phi_c$ (mm)	Type 1	Type 2	Type 3	Type 4	Type 5
$R_b$ ( $\mu\text{m}$ )	303.00	347.00	382.00	481.00	551.00
Average duration of path (s)	11.64	11.12	10.26	8.77	7.12
Average speed ( $\text{cm sec}^{-1}$ )	8.50	9.00	9.70	11.40	14.00
Bubble radius ( $\mu\text{m}$ )	327.00	342.00	364.00	412.00	484.00

$R_b$ : The radius calculated by Tate's law, at the orifice from where bubbles spring out

Average radii of bubbles are estimated according to Tate's law (to be sure that sizes of generated bubbles stay in the range 70-500  $\mu\text{m}$ ) and using results of Comolet<sup>[16]</sup> and Peebles<sup>[17]</sup> studies on the motion of an air bubble in a liquid. The results are assembled in Table 2.

**Acoustic response:** The transducer is immersed at 20 cm deep (distance free water surface-vibrating base of the transducer). This depth is kept constant by water bringing. The setting parameters of the generator and the oscilloscope have been chosen identical for all the recordings, in order to get the best relation signal/noise. The transducer is placed in such a way that its axis merges with the axis of the water column. So the strongest signal is recorded as bubble response, since the reflection coefficients of the bubbles surface are maximized<sup>[18,19]</sup>.

The oscilloscope allows a continuous vision of the response of the bubbles and the background noise. But as bubbles are a very good acoustic reflector, the information coming from the bubbles can be easily discerned from the background noise<sup>[20,21]</sup>.

As bubbles rise one after another, the recording works only when the distance between two following bubbles is big enough so that the reflecting volume can contain only one bubble. Thus each peak on the recording is compared to only one bubble response. With only one recording, there are several peaks, therefore several bubbles echoes. The time between the starting signal and one peak allows to calculate the distance  $z$  between the bubble and the transducer.

Figure 2 and 3 show respectively a complete recording and a bubble echo example.

**Statistical analysis:** Before developing analysis, let us give a brief reminder on acoustic wave attenuation in a liquid medium.

Every point in the acoustic field receives alternatively kinetic energy of moving particles and potential energy coming from pressure differences. The acoustic wave intensity decreases with the transducer-bubble distance because of the wave divergence and the attenuation that it gets by absorption

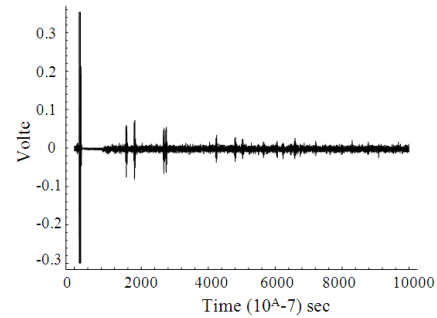


Fig. 2: A recording example where we see a first arch of very great power representing the excitation signal, then, others weaker in power, representing acoustic responses of bubbles

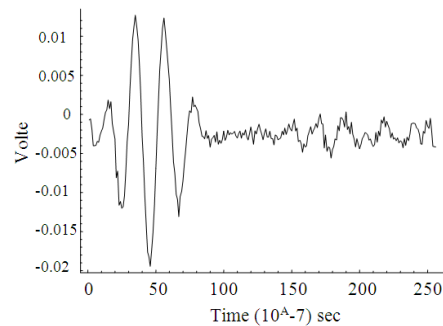


Fig. 3: An example of acoustic response of air bubble in the propagation medium. The divergence in an ideal medium can be defined<sup>[20]</sup> as:

$$I_r = \frac{I_0}{r^2}$$

where,  $I_0$  and  $I_r$  are the radiated acoustic intensities, respectively to the source and to the distance  $r$  from the source. In a real medium, the wave is attenuated because of the frictions between particles and this effect, taken separately, results in an exponential decrease of the acoustic intensity with the distance to the source. It is expressed by  $\exp(-\beta r)$  with a rate  $\beta$  depending on the acoustic characteristics of the medium and on the sound wavelength. Gathering both effects, this expression is defined<sup>[22,23]</sup> as:

$$I_r = \frac{I_0}{r^2} \exp(-\beta r)$$

By analogy with what has just been reminded, here is the description of the variations of the energy backscattered by a bubble in our experimental conditions:

$$E = \alpha \exp(-\beta z) \tag{1}$$

where, E represents the energy backscattered by a bubble situated at a distance z from the vibrating transducer base  $\alpha$  and  $\beta$  are two real coefficients.

The aim is to study the behavior of the two coefficients  $\alpha$  and  $\beta$  in relation to our very special experimental conditions: Water column (1 m), depth of the nearby field (calculated as 13 cm), very high work frequencies (500±200 kHz). It will be particularly useful with these special experimental conditions, to calculate coefficient  $\beta$  representing the attenuation by absorption.

Various calculus and analyses will be made with help of SAS programs and in particular its GLM procedure<sup>[24]</sup>.

**Preliminary analysis:**

**The data:** To study the energy backscattered by bubbles, data are taken from seventeen recordings (Table 3). They are filtered in order to eliminate background noise. The filter is a pass-band (450 kHz wide) and has been chosen according to the pass-band of the transducer. The result of this filtering is satisfactory as the signals corresponding to the bubbles are not at all damaged.

The integration of all the signals contained in each recording is performed after rectification of the negative part (calculus of the norm L1). The energizing contributions are calculated and each one will represent a chosen bubble. They are gathered according to bubbles size and form the data to analyze in function to depth.

**Correlations with size and depth:** Basic statistics on energy contributions of Monobubbles such as average , correlation (Pearson's) between energy and depth, are calculated in Table 4.

The average energy backscattered by a bubble increases according to its size, apart from type 4 bubbles (R = 410 μm). The number of bubbles and their relatively high depths would explain this singularity (Table 4).

The correlation coefficients explain the link between the variables: So, for the first bubble group (R = 325 μm) this coefficient (-0.84) expresses that the energy variations are up to 84% due to depth. The negative sign gives the direction of the variation and in this last case, it indicates that the more the depth increases, the more the energy backscattered by a bubble decreases: The smaller the bubble is, the more the energy variations that it sends back are controlled by its depth.

Table 3: Data represented by recordings and echoes numbers

Bubble type	1	2	3	4	5
Bubble radius (μm)	325	345	365	410	485
Recordings number	3	4	4	4	2
Number of observed echoes (bubbles)	10	21	41	47	29

**Note:** For each type of bubbles, a definite number of recordings was used. The number of bubble echoes appearing in each recording is different from one to another

Table 4: Statistics on energies backscattered by bubbles of distinct sizes

Bubble radius (μm)	325.00	345.00	365.00	410.00	485.00
Bubbles number	10.00	21.00	40.00	47.00	29.00
Average energy/ bubble×10 <sup>-7</sup> (Vs)	0.69	0.71	0.89	0.57	0.89
Correlation between energy and depth	-0.84	-0.66	-0.63	-0.57	-0.52

**Note:** The average value increases with the bubble size except in the fourth case. The correlation coefficients decrease (in absolute value) with bubble size

The values of correlation coefficients "energy-depth" and "energy-bubbles size" are respectively -0.55 and -0.03 in the case where all data are gathered without size distinction. The fact that the global correlation between energy and size is very weak shows that size has not got a direct relation with energy.

The results gathered in Table 4 show that size influences the shape (and/or intensity) of the relation between energy and depth.

**Regressions on data according to size:** For each size of bubble, a regression according to an exponential model energy-depth is performed. In practice the regression is calculated between log(energy) and log(depth). This elementary transformation allows to stay in the domain of linear analysis. Applied to data, regression allows an adjustment by exponential curves whose general expression is as relation (1). The results summarized on Fig. 4 show that apparently the coefficients  $\alpha$  and  $\beta$  vary according to the bubbles size. Then the following questions may be asked:

- Is this variation significant?
- Has depth an effect on  $\alpha$  or  $\beta$ ? In other words, has the model 1 a local or global validity?

**Study of energy-depth curves:**

**Variance and covariance analysis:** The coefficients  $\alpha$  and  $\beta$  will be connected to both factors controlling energy variations: Size and depth. In order to test the effect of these factors, analysis of variance is usually employed. For this a third variable will be introduced, substituting for depth (non qualitative variable), that will be called "layer" and which has the real signification of a liquid layer. This new variable have four modalities:

- Layer A-7 cm ≤ z ≤ 15.6 cm
- Layer B-15.6 cm < z < 31.3 cm
- Layer C-31.3 cm ≤ z < 45.3 cm
- Layer D-z ≥ 45.3 cm

The number of layers is chosen in such a way that the number of bubbles appearing in each of them has the same magnitude. In addition, the fact that layer A is partly situated in the near field transducer is aimed to see if it will behave differently from the others.

First, the interpretation of the energy variations in function to both factors (size and layer) is realized by an analysis of variance according to the following not linear model<sup>[25]</sup>:

$$E_{t,c} \equiv E_{moy} + E_t + E_c + E_{t*c} + \varepsilon \quad (2)$$

This equation means that the energy  $E_{t,c}$  associated to bubbles of size  $t$ , insonified in layer  $C$ , expresses like the sum of an average term  $E_{moy}$ , plus a term linked to size  $E_t$ , plus a term linked to layer  $E_c$ , plus a term linked to the interaction size-layer  $E_{t*c}$  and plus a term representing the error  $\varepsilon$ .

The results of this analysis show that:

- terms  $E_t$  and  $E_c$  are significant
- the interaction size-layer term is not significant:  $E_{t*c} = 0$

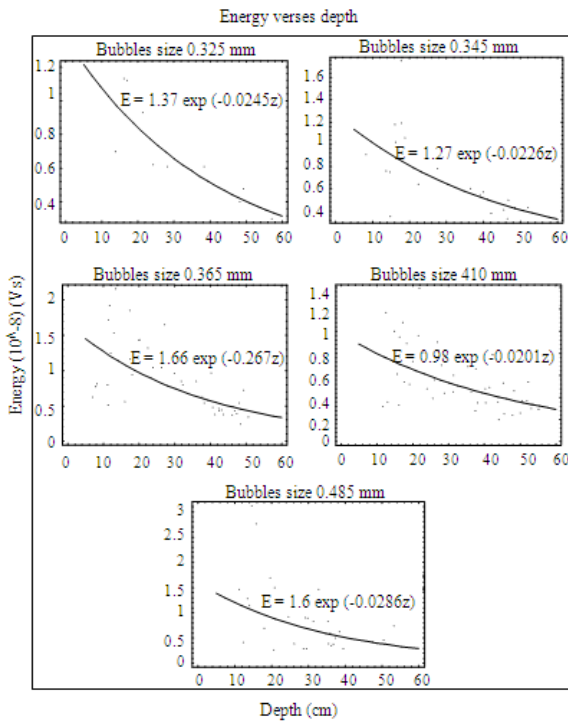


Fig. 4: Adjustment of energy-depth curves for each type of bubbles

Which reduces the model equation to:

$$E_{t,c} \equiv E_{moy} + E_t + E_c + \varepsilon \quad (3)$$

In conclusion, for each bubble size, the dependence between the energy ( $\log E$ ) and the layer is a step function. Can dependence express itself by means of a linear model?

To answer this question, depth  $z$  is re-introduced as a new active variable and, through a covariance analysis, a mixed model<sup>[26]</sup> is tested, as follows:

$$E_{t,c} \equiv E_{moy} + E_t + E_c + \beta z + \varepsilon \quad (4)$$

$\beta z$  represents here a linear effect of depth, if need be completed by non-linear term  $E_c$ .

The results of this last analysis show that:

- Effect layer becomes non significant  $E_c \equiv 0$
- Effect size remains significant
- $\beta$  is significantly different from zero

The disappearance of effect layer shows that the linear model is satisfactory enough to realize the part of depth.

Finally, the model is reduced to:

$$E_{t,z} \equiv E_{moy} + E_t + \beta z + \varepsilon \quad (5)$$

If  $\tilde{E}_t = E_{moy} + E_t$ ,  $E_{t,z}$  becomes:

$$E_{t,z} \equiv \tilde{E}_t + \beta z + \varepsilon$$

As  $E_{t,z}$  represents in fact  $\log(E)$ , the transition to exponential gives:

$$\exp(E_{t,z}) = \exp(\tilde{E}_t + \beta z + \varepsilon)$$

If the term "error" is very small ( $\varepsilon \equiv 0$ ) and if  $\exp(\tilde{E}_t) = \alpha_t$ , we obtain:

$$E(t,z) = \alpha_t \exp(\beta z) \quad (6)$$

The theoretical model proposed at the beginning (1) is back again with:

- A coefficient  $\alpha$  which depends on bubbles size (or on experimental conditions)
- A coefficient  $\beta$  which is constant

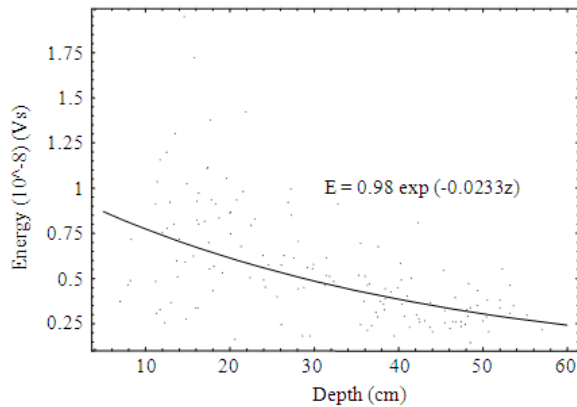


Fig. 5: Adjustment of energy-depth curve for all gathered bubble

As a deduction effect "size" represented by coefficient  $\alpha$  is very significant and the fluctuations of coefficient  $\beta$  are negligible.

**Estimation of absorption coefficient:** Coefficient  $\beta$  being constant in the five studied cases, the data set (without size distinction) is submitted to a general regression, after correction by coefficients  $\alpha_r$ .

In each data group,  $E$  is divided by  $\alpha$ , then data are gathered together. An adjustment at the sense of the least squares gives the results shown on Fig. 5.

The numerical values calculated for  $\alpha$  and  $\beta$  when  $E$  is expressed in  $10^{-8}$  Vs and  $z$  in cm, are:

$$\alpha = 0.98 \text{ and } \beta = -0.0233$$

The homogeneity of the corrected data is confirmed by the fact that  $\alpha$  is close to unit ( $\alpha \cong 1$ ).

If  $E_0$  is the source energy, it is for all bubbles without size distinction:

$$\frac{E}{E_0} = \exp(-0.0233z) \tag{7}$$

In conclusion, the energy backscattered by a bubble varies with depth according to the model (1) where coefficient  $\alpha$  depends on the bubble size and  $\beta$  is constant and linked only to the experimental medium.

### RESULTS

The first result of the starting study is the realization of the bench test. Indeed, the resemblance of the bench with the real middle, especially the techniques of production of calibrated bubbles in a

liquid middle, is so much surmounted with difficulties. The fact that these bubbles appear in the column of water, one by one (where from the naming of Monobubbles), allowed the recording of a signal resulting from a unique diffuser.

Because of the differences very marked the acoustic impedances between air and water, the air bubbles are known to be excellent diffusers of the acoustic signals. Nevertheless, the acousticians always studied them in the resonance frequency domain to obtain the best possible acoustic answer. By custom, it became a current and usual principle when we deal as air bubbles.

Our study, driven outside resonance domain of air bubbles, denied this principle because the acoustic answers of the obtained bubbles are easily localizable on the recordings. We can even advise to sound the aquatic middles where there are many air bubbles outside resonance to avoid saturating the recordings.

In the part dedicated to the signal processing of bubbles, we showed that:

- The non linear model to explain the variations of the backscattered energy by an air bubble was useless; these variations are very described by a linear model
- The effect "bubble size" was connected to coefficient  $\alpha$  appearing in the expression of the reserved linear model
- the absorption is, itself, represented by a coefficient  $\beta$  expressing the proportionality with the depth
- The absorption coefficient  $\beta$  was estimated in our experimental conditions

### DISCUSSION

The realized bench test is functional, even if we feigned the real conditions offshore, especially in the absence of current and other diffusers that bubbles in the environment. However, it is practicable and easily adaptable.

Concerning production of calibrated bubbles, it remains to study the possibility of generating different sizes bubbles, present at the same time in the water column. In that case, the finalized statistical model could allow discriminating between the acoustic signals by the size of bubbles having backscattered them.

In this study, the air bubbles radii were calculated from the ascent limit speed of bubbles. But, when we know that bubbles appear in the water column by series like rosaries, we can wonder about the updraft of the rosary and its influence on the speed of bubbles. This

flow can thus modify bubbles speeds and consequently induce differences between the calculated radii and the real radii of bubbles. For us, we think that this flow is very weak and that its influence on the speed of bubbles is limited as in our case all happens on a small distance of hardly 1 m.

The finalized statistical model will, on the other hand, allow to determine on a recording the depth of the bubble having backscattered this signal. It constitutes a considerable advance in this research subject.

## CONCLUSION

At the end, we succeeded to build a pilot study which simulates approximately the offshore experimental conditions: Acoustic sounding of a water column from the surface and with a single transducer acting simultaneously as transmitter and as receptor.

On the other part, we made an original technique for the production of calibrated air bubbles in a liquid environment and we have recorded a signal backscattered by a unique bubble (Monobubbles) of known size.

The first statistical treatments allowed determining the attenuation of this signal in the environment water-air bubbles.

The coming step will be to correct the signal of the effect of the absorption of energy by the middle, then to elaborate a protocol of localization of the signals of bubbles on recordings where multiple diffusers appear by their acoustic answers.

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