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Reliability Equivalence Factors of a System with m Non-Identical Mixed of Lifetimes

[1]Abdelfattah Mustafa and [2]Adel A. El-Faheem
[1]Department of Mathematics, College of Science and Humanitarian Studies, Al-Kharj, University, P.O. 83, Al-Kharj 11942, Saudi Arabia
[2]Department of Mathematics, Faculty of Science, South Valley University, Aswan, Egypt

Abstract: Problem statement: The aim of this study is to generalize reliability equivalence technique to apply it to a system consists of m independent and non-identical lifetimes distributions, with mixed failure lifetimes $f_1(t), f_2(t), \ldots, f_m(t)$. **Approach:** We shall improve the system by using some reliability techniques: (i) reducing the failure for some lifetimes; (ii) add hot duplication component; (iii) add cold duplication component; (iv) add cold duplication component with imperfect switch. We start by establishing two different types of reliability equivalence factors, the Survival Reliability Equivalence (SRE) and Mean Reliability Equivalence (MRE) factors. Also, we introduced a numerical illustrative example. **Results:** The system reliability function and mean time to failure will be used as reference of the system performances. For this reason, we obtain the reliability functions and mean time to failures of the original and improved systems using each improving methods. **Conclusion:** The results can be used to distinguish between the original and improved systems performances and calculate the equivalent between different cases of improving methods.

Key words: Mixture distributions, reliability equivalence, improving system, exponential distribution, numerical illustrative, equivalence factors, theoretical results, reliability functions, duplication method, Mean Reliability Equivalence (MRE)

INTRODUCTION

The concept of the reliability equivalence factors introduced by (Rade (1993a, 1993b)) was applied of simple series and parallel systems consists of one or two components. Later, Sarhan (2000, 2002), Mustafa (2009) applied the same concept on more general and complex systems. The reliability equivalence factors of the system is that factors $\rho, 0< \rho<1$ by which the failure rates of some system components should be reduced to get a reliability for the system as that for a system obtained by assuming the improved methods mentioned above. We consider a system component with mixing lifetimes $f_1(t), f_2(t), \ldots, f_m(t)$, the density function for this system can be write as follows, Everitt and Hand (1981), Akay (2007) and Teamah and El-Bar (2009):

$$f(t) = \sum_{i=1}^{m} p_i f_i(t), 0 \le p_i \le 1, \sum_{i=1}^{m} p_i = 1 \quad (1.1)$$

Then, the reliability function becomes:

$$R(t) = \sum_{i=1}^{m} p_i R_i(t) \quad (1.2)$$

and the failure rate can be obtained as:

$$h(t) = \frac{f(t)}{R(t)} = \frac{\sum_{i=1}^{m} p_i f_i(t)}{\sum_{i=1}^{m} p_i R_i(t)} \quad (1.3)$$

The original system: We derive the reliability and mean time to failure for the system with the mixing lifetime distribution. Assuming any mixed has the constant failure rate, λ_i, i = 1, 2, …, m, Abu-Taleb *et al*. (2007), Al-Kutubi and Ibrahim (2009), that is:

$$f_i(t) = \lambda_i \exp\{-\lambda_i t\}, i = 1, 2, .., m$$

The functions f(t), R(t), can be obtained as follows, Jamjoom and AL-Saiary (2010):

$$f(t) = \sum_{i=1}^{m} p_i \lambda_i \exp\{-\lambda_i t\} \quad (2.1)$$

Corresponding Author: A. Mustafa, Department of Mathematics, College of Science and Humanitarian Studies, Al-Kharj, University, P.O. 83, Al-Kharj 11942, Saudi Arabia

$$R(t) = \sum_{i=1}^{m} p_i \exp\{-\lambda_i t\} \qquad (2.2)$$

From equation (2.2), one can easily obtain the mean time to failure, say MTTF as follows Billinton and Allan (1992), Zio (2007) and Rushdi and Alsulami (2007):

$$MTTF = \int_0^{\infty} R(t)dt = \sum_{i=1}^{m} \frac{p_i}{\lambda_i} \qquad (2.3)$$

The improved systems: The quality of the system reliability can be improved using four different methods of the system improvements, Haggag (2009).

Reduction method: Let $R_{A,\rho}(t)$ denotes the reliability function of the improved system when some the failure rate of the set A, of mixing components are reduced by the factor $\rho_i, 0<\rho_i<1$. One can obtain the function $R_{A,\rho}(t)$, as follows:

$$R_{A,\rho}(t) = \sum_{i \in A} p_i \exp\{-\rho_i \lambda_i t\} + \sum_{i \in \bar{A}} p_i \exp\{-\lambda_i t\} \qquad (3.1)$$

where, $|A| = r, |\bar{A}| = m - r$.

From Eq. 3.1, the mean time to failure of the improved system, say $MTTF_{A,\rho}$, becomes:

$$MTTF_{A,\rho} = \sum_{i \in A} \frac{p_i}{\rho_i \lambda_i} + \sum_{i \in \bar{A}} \frac{p_i}{\lambda_i} \qquad (3.2)$$

We can rewrite the $MTTF_{A,\rho}$, in the following form:

$$MTTF_{A,\rho} = MTTF + \sum_{i \in A} \frac{(1-\rho_i)p_i}{\rho_i \lambda_i} \qquad (3.3)$$

That is, reduction method of a set A of mixing components increases the mean time to failure by the amount $\sum_{i \in A} \frac{(1-\rho_i)p_i}{\rho_i \lambda_i}$.

Hot duplication method: Let $R^H(t)$ be the reliability function of the improved system obtained by assuming hot duplications of a component. The function $R^H(t)$ is given by, Lewis (1996) and Birolini (2007):

$$R^H(t) = \left[2 - \sum_{i=1}^{m} p_i \exp\{-\lambda_i t\}\right] \sum_{i=1}^{m} p_i \exp(-\lambda_i t) \qquad (3.4)$$

Let $MTTF^H$ be the mean time to failure of improved system assuming hot duplication method. Using Eq. 3.4, one can deduce $MTTF^H$ as:

$$MTTF^H = \sum_{i=1}^{m} p_i \left[\frac{2}{\lambda_i} - \sum_{j=1}^{m} \frac{p_j}{\lambda_i + \lambda_j}\right] \qquad (3.5)$$

We can rewrite the $MTTF^H$ in the following form:

$$MTTF^H = MTTF + \sum_{i=1}^{m} p_i \left[\frac{1}{\lambda_i} - \sum_{j=1}^{m} \frac{p_j}{\lambda_i + \lambda_j}\right]$$

That is, hot duplication of a single component increases the mean time to system failure by the amount $\sum_{i=1}^{m} p_i \left[\frac{1}{\lambda_i} - \sum_{j=1}^{m} \frac{p_j}{\lambda_i + \lambda_j}\right]$.

Cold duplication method: Let $R^C(t)$ be the reliability function of the improved system obtained by assuming cold duplications of the system component. The function $R^C(t)$ can be obtained as follows:

$$R^C(t) = \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{p_i p_j}{\lambda_i - \lambda_j} [\lambda_i \exp\{-\lambda_j t\}] - \lambda_j \exp\{-\lambda_i t\} \qquad (3.6)$$

From equation (3.6), the mean time to failure of the improved system, say $MTTF^C$, assuming cold duplication method is given as:

$$MTTF^C = \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{p_i p_j (\lambda_i + \lambda_j)}{\lambda_i \lambda_j} \qquad (3.7)$$

We can rewrite the $MTTF^C$ in the following form:

$$MTTF^C = MTTF + \sum_{i=1}^{m} \frac{p_i}{\lambda_i} \left[\sum_{j=1}^{m} \frac{p_j(\lambda_i + \lambda_j)}{\lambda_j} - 1\right]$$

That is, cold duplication of the system component increases the mean time to system failure by the amount $\sum_{i=1}^{m} \frac{p_i}{\lambda_i} \left[\sum_{j=1}^{m} \frac{p_j(\lambda_i + \lambda_j)}{\lambda_j} - 1\right]$.

Imperfect switching duplication method: Let us consider now that, the system reliability can be improved assuming cold duplication method with imperfect switch of the system component. In such method, it is assumed that the component is connected by a cold redundant standby component via a random switch having a constant failure rate, say β.

Let $R^I(t)$ be the reliability function of the improved system when the system component is improved according to the cold duplication method with imperfect switch. The function $R^I(t)$, is given as follows:

$$R^{I}(t)=\sum_{i=1}^{m}\sum_{j=1}^{m}p_i p_j\lambda_i\left[\begin{array}{l}\dfrac{\lambda_j}{\lambda_i-\lambda_j+\beta}\left(\begin{array}{l}\dfrac{\exp\{-\lambda_j t\}}{\lambda_j}\\-\dfrac{\exp\{-(\lambda_i+\beta)t\}}{\lambda_i+\beta}\end{array}\right)\\+\dfrac{\beta}{\lambda_i-\lambda_j-\beta}\left(\begin{array}{l}\dfrac{\exp\{-(\lambda_j+\beta)t\}}{\lambda_j+\beta}\\-\dfrac{\exp\{-\lambda_i t\}}{\lambda_i}\end{array}\right)\end{array}\right] \quad (3.8)$$

From Eq. (3.8), the mean time to failure of the improved system, say $MTTF^I$ is given by:

$$MTTF^{I}(t)=\sum_{i=1}^{m}\sum_{j=1}^{m}p_i p_j(\lambda_i+\lambda_j+\beta)\left[\frac{\lambda_i}{\lambda_j(\lambda_i+\beta)^2}+\frac{\beta}{\lambda_i(\lambda_j+\beta)^2}\right] \quad (3.9)$$

We can rewrite the $MTTF^I$, as the following form:

$$MTTF^{I}(t)=MTTF+\sum_{i=1}^{m}p_i\left\{\begin{array}{l}\sum_{j=1}^{m}p_j(\lambda_i+\lambda_j+\beta)\times\\\left[\dfrac{\lambda_i}{\lambda_j(\lambda_i+\beta)^2}+\dfrac{\beta}{\lambda_i(\lambda_j+\beta)^2}\right]-\dfrac{1}{\lambda_i}\end{array}\right\}$$

That is, cold duplication with imperfect switch of the system component increases the mean time to system failure by the amount:

$$\sum_{i=1}^{m}p_i\left\{\sum_{j=1}^{m}p_j(\lambda_i+\lambda_j+\beta)\left[\frac{\lambda_i}{\lambda_j(\lambda_i+\beta)^2}+\frac{\beta}{\lambda_i(\lambda_j+\beta)^2}\right]-\frac{1}{\lambda_i}\right\}$$

MATERIALS AND METHODS

The α-fractiles: Let $L(\alpha)$ be the α-fractile of the original system and $L^D(\alpha)$, $D = H,C$ and I, the α-fractiles of the improved systems. The α-fractiles $L(\alpha)$ and $L^D(\alpha)$ are defined as the solution of the following equations, respectively:

$$R\left(\frac{L(\alpha)}{\Lambda}\right)=\alpha,\ R_A^D\left(\frac{L(\alpha)}{\Lambda}\right)=\alpha \quad (4.1)$$

where, $\Lambda=\sum_{i=1}^{m}\lambda_i$.

It follows from Eq. 2.2 and the first Eq. 4.1 that $L = L(\alpha)$, satisfies the following equation:

$$\sum_{i=1}^{m}p_i\exp\left\{-\frac{\lambda_i}{\Lambda}L\right\}=\alpha \quad (4.2)$$

From the second Eq. 4.1, when $D = H$ and Eq. 3.4, one can verify that $L = L^H(\alpha)$ satisfies the following equation:

$$\left[2-\sum_{i=1}^{m}p_i\exp\{-\frac{\lambda_i}{\Lambda}L\}\right]\sum_{i=1}^{m}p_i\exp\{-\frac{\lambda_i}{\Lambda}L\}=\alpha \quad (4.3)$$

Similarly, from Eq. 3.6 and the second Eq. 4.1, when $D = C$, $L = L^C(\alpha)$ can be obtained by solving the following equation:

$$\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{p_i p_j}{\lambda_i-\lambda_j}\left[\lambda_i\exp\{-\frac{\lambda_j}{\Lambda}L\}-\lambda_j\exp\{-\frac{\lambda_i}{\Lambda}L\}\right]=\alpha \quad (4.4)$$

Finally, from Eq. 3.8 and the second Eq. 4.1, when $D = I$, $L = L^I(\alpha)$ satisfies the following equation:

$$\sum_{i=1}^{m}\sum_{j=1}^{m}p_i p_j\lambda_i\left\{\begin{array}{l}\dfrac{\lambda_j}{\lambda_i-\lambda_j+\beta}\left[\begin{array}{l}\dfrac{1}{\lambda_j}\exp\left\{-\dfrac{\lambda_j}{\Lambda}L\right\}-\dfrac{1}{\lambda_i+\beta}\exp\\\left\{-\dfrac{(\lambda_i+\beta)}{\Lambda}L\right\}\end{array}\right]\\+\dfrac{\beta}{\lambda_i-\lambda_j-\beta}\left[\begin{array}{l}\dfrac{1}{\lambda_j+\beta}\exp\left\{-\dfrac{\lambda_j+\beta}{\Lambda}L\right\}\\-\dfrac{1}{\lambda_i}\exp\left\{-\dfrac{\lambda_i}{\Lambda}L\right\}\end{array}\right]\end{array}\right\} \quad (4.5)$$

Equation 4.2-4.5 have no closed form solutions and can be solved using some numerical program such as Mathematica Program System.

Reliability equivalence factors: We derive the SREF and MREF of the system component. Where we improve the system component according to one of the duplication methods (HDM, CDM and IDM) and A is the set of mixed lifetimes that be improved according to a reduction method.

The survival reliability equivalence factor: We shall derive the SREF, when the set A of mixing failure lifetime of the system component are reduced by the different factor ρ_i, $i = 1,2,\ldots,m$, these factors will be denoted by $\rho_i^D(\alpha)$, D=H,C,I and $i\in A, A\subseteq\{1,2,3,......,m\}$. The factor $\rho_i^D(\alpha)$ is defined as the solution ρ of the equation:

$$R^D(t) = R_{A,\rho}(t) = \alpha \quad (5.1)$$

Using Eq. 5.1, with Eq. 3.1 one can verify that the factor $\rho_i = \rho_i^D(\alpha)$ satisfies the following system of equations:

$$\left.\begin{array}{c}\sum_{i\in A} p_i \exp\{-\rho_i \lambda_i t\} + \sum_{i\in \bar{A}} p_i \exp\{-\lambda_i t\} = \alpha \\ R^D(t) = \alpha\end{array}\right\} \quad (5.2)$$

Equations system (5.2) have no closed form solutions and can be solved using some numerical program such as Mathematica Program System, when D = H,C,I, by using Eq. 3.4, 3.6 and 3.8.

The mean reliability equivalence factor: The MREF, say ξ_i^D for D = H,C,I and $i \in A, A \subset \{1,2,....,m\}$ can be obtained by solving the following equation:

$$MTTF_{A,\rho} = MTTF^D \quad (5.3)$$

Using Eq. 3.2 together with Eq. 5.3, one can verify that $\xi_i = \xi_i^D$ satisfies the equation:

$$\sum_{i\in A} \frac{(1-\xi_i)p_i}{\xi_i \lambda_i} = MTTF^D - MTTF \quad (5.4)$$

If we reduce the mixing failure rate for the lifetime by the same factor ξ, this means put $\xi_i = \xi$, for $i \in A$, we have:

$$\xi^D = \frac{\sum_{i\in A} \frac{p_i}{\lambda_i}}{MTTF^D - MTTF + \sum_{i\in A} \frac{p_i}{\lambda_i}} \quad (5.5)$$

Equation 5.4 can be solved numerically by using Mathematica program System, to get ξ^D for given A, m and λ_i. The $MTTF^D$ are given, for D = H,C and I, from Eq. 3.5, 3.7 and 3.9 respectively.

RESULTS

To explain how one can utilize the previously obtained theoretical results, we introduce a numerical example.
In such example, we calculate the two different reliability equivalence factors of a system of one component with three-mixing lifetime that is m = 3, under the following assumptions:

- The failure rates of the mixing lifetime i, i = 1,2,3, are $\lambda_1 = 0.09$, $\lambda_2 = 0.07$ and $\lambda_3 = 0.08$
- The probability vector p is p = (0.4, 0.35, 0.25).
- The system reliability will be improved when the system component is improved according to one of the previous duplication methods.
- In the reduction method, we improve the system reliability when one; two or three types of mixing lifetime are reducing by the factor ρ.
- In the imperfect switch duplication method β = 0.04

Table 1: The MTTF of the original and improved systems

MTTF	$MTTF^H$	$MTTF^I$	$MTTF^C$
12.5694	18.8912	20.9115	25.1389

Table 2:The α-fractiles of the original and improved system

α	L(α)	L^H	L^I	L^C
0.1	6.9585	9.0103	9.8453	11.7753
0.2	4.8438	6.7924	7.5116	9.0325
0.3	3.6148	5.4597	6.0856	7.3419
0.4	2.7464	4.4807	5.0248	6.0771
0.5	2.0749	3.6873	4.1562	5.0366
0.6	1.5275	3.0015	3.3985	4.1256
0.7	1.0656	2.3765	2.7022	3.2856
0.8	0.6661	1.7734	2.0247	2.4657
0.9	0.3143	1.1358	1.3025	1.5888

Table 3: The SREF ρ_A^D

	A = {1}			A = {1,2}			A = {1,2,3}		
α	ρ^H	ρ^I	ρ^C	ρ^H	ρ^I	ρ^C	ρ^H	ρ^I	ρ^C
0.1	0.5503	0.4690	0.3540	0.7211	0.6494	0.5304	0.7723	0.7068	0.5909
0.2	0.4543	0.3716	0.2658	0.6472	0.5726	0.4613	0.7131	0.6448	0.5363
0.3	0.3680	0.2869	0.1897	0.5832	0.5088	0.4047	0.6621	0.5939	0.4924
0.4	0.2818	0.2040	0.1153	0.5209	0.4485	0.3518	0.6129	0.5466	0.4519
0.5	0.1903	0.1170	0.0373	0.4568	0.3875	0.2985	0.5627	0.4992	0.4119
0.6	0.0881	0.0207	NA	0.3872	0.3224	0.2418	0.5089	0.4495	0.3702
0.7	NA	NA	NA	0.3078	0.2490	0.1779	0.4484	0.3943	0.3243
0.8	NA	NA	NA	0.2107	0.1599	0.1005	0.3756	0.3290	0.2702
0.9	NA	NA	NA	0.0751	0.0367	0.0079	0.2767	0.2413	0.1978

Table 4: The MREF, ξ_A^D

A	ξ^H	ξ^I	ξ^C
{1}	0.412813	0.347588	0.261224
{1,2}	0.599029	0.530988	0.429021
{1,2,3}	0.665358	0.601077	0.499999

For this example, we have found that:

The mean time to failure of the original system and improved systems assuming hot, cold, imperfect switch duplication methods are presented in Table 1.

The α-fractiles $L(\alpha), L^D(\alpha)$ and the reliability equivalence factors $\rho_A^D(\alpha), D = H, C, I$ and $A \subseteq \{1,2,3\}$ are calculated using Mathematica Program System according to the previous theoretical formulae. In such calculations the level α is chosen to be 0.1,0.2,…,0.9.

Table 2 represents the α-fractiles of the original and improved systems that are obtained by improving the system component according to the previously mentioned methods.

Table 3 shows the survival reliability equivalence factor of the improved systems using each duplication method for some A.

Table 4 shows the mean reliability equivalence factor of the improved systems using each duplication method for some A.

DISCUSSION

From Table 1, one can conclude that:

$$MTTF < MTTF^H < MTTF^I < MTTF^C$$

Based on the results presented in Table 2, it seems that:

$L(\alpha) < L^H(\alpha) < L^I(\alpha) < L^C(\alpha)$ in all studied cases.

This is confirmed by the results obtained for MTTF.

According to the results presented in Table 3, it may be observed that:

- Hot duplication of the system component, will increase L(0.1) from $\frac{6.9585}{\Lambda}$ to $\frac{9.0103}{\Lambda}$, see Table 2. The same effect on L(0.1) can occur by reducing the failure rates of mixing lifetimes of (i) type one, A = {1}, by the factor ρ = 0.550.3, (ii) types one and two, A = {1,2}, by the factor ρ = 0.7211, (iii) three types, A = {1,2,3}, by the factor ρ = 0.7723, see Table 3.
- Imperfect switch duplication of the system component, will increase L(0.1) from $\frac{6.9585}{\Lambda}$ to $\frac{9.8453}{\Lambda}$, see Table 2. The same effect on L(0.1) can occur by reducing the failure rates of mixing lifetimes of (i) type one, A = {1}, by the factor ρ = 0.4690, (ii) types one and two, A = {1,2}, by the factor ρ = 0.6494, (iii) three types, A = {1,2,3}, by the factor ρ = 0.7068, see Table 3.
- Cold duplication of the system component, will increase L(0.1) from $\frac{6.9585}{\Lambda}$ to $\frac{11.7753}{\Lambda}$, Table 2. The same effect on L(0.1) can occur by reducing the failure rates of mixing lifetimes of (i) type one, A = {1}, by the factor ρ = 0.3540, (ii) types one and two, A = {1,2}, by the factor ρ = 0.5304, (iii) three types, A = {1,2,3}, by the factor ρ = 0.5909, see Table 3.
- In the same manner, one can read the rest of results presented in Table 3.
- The notation NA, means that there is no equivalence between the two improved systems: one obtained by reducing the failure rates of the set A of the system components and the other obtained by improving the system component according to the duplication methods

Based on the results presented in Table 4, one can conclude that:

- The improved system that can be obtained by improving the system component, according to hot duplication method, has the same mean time to failure of that system which can be obtained by doing one of the following (i) reducing the failure rate of type 1 of mixing lifetime, A = {1}, by the factor $\xi = 0.4128$, (ii) reducing the failure rates of type 1,2 of mixing lifetime, A = {1,2}, by the factor $\xi = 0.5990$, (iii) reducing the failure rates of three types of mixing lifetime, A = {1,2,3}, by the factor $\xi = 0.6654$, see Table 4
- In the same manner, one can read the rest of results presented in Table 4, when the other duplication methods are used with different A

CONCLUSION

The quality of the system reliability can be improved using four different methods of the system improvements. The results can be used to distinguish between the original and improved systems performances and calculate the equivalent between different cases of improving methods.

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